Lab 3

January 27, 2017

1 Chapter 6.1.2

Question: Show that a profit-maximizing monopolist's output is unaffected by a proportional profit tax, but is reduced by a tax of $t per unit of output. Explain these results.

Hints: Let's assume this monopolist following cost function $C(q)$.

If no tax, profit maximization problem (PMP) for the monopoly is:

$$Max: \ \pi = p * q - C(q) \tag{1}$$

Differentiating π w.r.t q, we can implicitly find the optimal output level q^*, such that:

$$FOC: p'(q^*) * q^* + p(q^*) - C'(q^*) = 0 \tag{2}$$

In order to have a maximum, the second order has to satisfy:

$$SOC: p'(q) + p''(q) * q + p'(q) - C''(q) < 0 \tag{3}$$

Now, with a **proportional** profit tax, say, t. The PMP for the monopoly becomes:

$$Max: \ \pi = (1-t)(p * q - C(q)) \tag{4}$$

Differentiating π w.r.t q, we can implicitly find the optimal output level q^*.

$$FOC: (1-t)(p'(q^*) * q^* + p(q^*) - C'(q^*)) = 0 \tag{5}$$

You may notice that, if we divide (4) both sides by $(1-t)$, we will get the same answer as in (2) (no tax case). This means the optimal output level q^* is UNRELATED to the tax t, which implies the monopolist's output is unaffected by a proportional profit tax. However, now with a **unit** tax $\$t$, The PMP for the monopoly becomes:

$$Max: \ \pi = p * q - C(q) - t * q \tag{6}$$

Differentiating π w.r.t q, we can implicitly find the optimal output level q^*.

$$FOC: p'(q^*) * q^* + p(q^*) - C'(q^*) - t = 0 \tag{7}$$

From (6), we can see that the optimal output level, q^*, actually is a function of unit tax t. This implies with t changes, the optimal output level has to change.

Since we know that (when we solved (7)) q^* is a function of t, we can rewrite (7) as :

$$FOC: p'(q^*(t)) * q^*(t) + p(q^*(t)) - C'(q^*(t)) - t = 0 \tag{8}$$

Now, let's differentiate (8) w.r.t t (this is called comparative static analysis in economics), we have:

$$\frac{dq^*}{dt} * (p'(q^*) + p''(q^*) * q^* + p'(q^*) - C''(q^*)) - 1 = 0 \tag{9}$$

$$\frac{dq^*}{dt} = \frac{1}{p'(q^*) + p''(q^*) * q^* + p'(q^*) - C''(q^*)} \tag{10}$$

Following (3), we can determine that:

$$\frac{dq^*}{dt} = \frac{1}{p'(q^*) + p''(q^*) * q^* + p'(q^*) - C''(q^*)} < 0 \tag{11}$$

This means that with unit tax increase, the optimal output level will be reduced.

Intuitively, this is because the proportional profit tax is a **lump sum** tax. It will not distort the market and the optimal choice of the monopoly will not be changed. However, the unit tax is a **distortionary** tax. It distorts the market like an **externality**, which lead monopoly to change the output based on its own maximization problem.

2 Chapter 6.2.2

Question: A firm in a competitive market discovers a new production process which gives it the total-cost function

$$C = 10 * logx \tag{12}$$

where x is output. Explain as fully as you can, in both mathematical and economic terms, why there may be a breakdown of perfect competition in this market.

Hints: Page 205 - 206 of your text book (3^{rd} edition) gives you the proof that why a firm with a linear cost function cannot support a competitive equilibrium. Same arguments apply to this problem.

Let's characterize the properties of our cost function first:

Differentiating C w.r.t x, we have:

$$\frac{dC}{dx} = 10 * \frac{1}{x} > 0 \tag{13}$$

Differentiating C w.r.t x *TWICE*, we have:

$$\frac{d^2C}{dx^2} = -10 * \frac{1}{x^2} < 0 \tag{14}$$

Based on (13) & (14), we find that the cost function is increasing and concave in x.

Since we consider a competitive market, the market price p is given and **cannot** be manipulated by the firm. Now, assuming that this competitive market has an equilibrium, we must have:

$$p = marginal\ cost = 10 * \frac{1}{x^*} \tag{15}$$

And the firm has profit:

$$\pi = px^* - 10 * logx^* = 0 \tag{16}$$

However, this firm still has the incentive to deviate from this equilibrium. To see this, let's consider the firm produce x^*+1 units of output. The additional output gives firm p additional revenue and incurs $10 * \frac{1}{x^*+1}$ additional cost. We must have:

$$p = marginal\ cost = 10 * \frac{1}{x^*} > 10 * \frac{1}{x^* + 1} \tag{17}$$

This means the additional unit gives this firm additional positive profit. And since the marginal cost is always decreasing, the firm would always have incentive to deviate from this equilibrium and produce more output infinitely, which means there is no profit maximization output level.

Intuitively, this is because this firm is **increasing return to scale**

3 Chapter 6.2.3

Question: A monopolist has the inverse demand function

$$p = a - b * x \tag{18}$$

and the total-cost function

$$C = 10 * log x \tag{19}$$

Give conditions under which there will be a well-defined, profit-maximizing output and explain your answer in a diagram.

Solution: Check page 932 of your text book (3^{rd} edition)