Empirical Panel Data: Lecture 10

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Topic 4: An empirical example for the dynamic panel regression

- Recall, we use the following model as an example before

$$\Delta G_{it} = \beta_0 + \beta_1 \Delta C_{it} + \beta_2 \Delta K_{it} + \beta_3 \Delta P_{it} + \alpha_i + \varepsilon_{it}.$$

- ΔG_{it} is the economic growth rate
- ΔC_{it} is the consumption growth rate
- ΔK_{it} is the investment growth rate
- ΔP_{it} is the population growth rate

- Since the output level is essentially persistent, we may expect the empirical growth model should allow this dynamic process. Therefore, we extend above static panel model to the augmented dynamic panel AR(1) panel:

$$\Delta G_{it} = \beta_0 + \gamma \Delta G_{it-1} + \beta_1 \Delta C_{it} + \beta_2 \Delta K_{it} + \beta_3 \Delta P_{it} + \alpha_i + \varepsilon_{it}.$$

A Stata example: Fixed effect estimation without IV

- xtreg lrgdpnagrowth l.lrgdpnagrowth lccongrowth lckgrowth lpopgrowth, fe

```
Fixed-effects (within) regression               Number of obs     =      9,050
Group variable: country1                        Number of groups  =        180

R-sq:                                           Obs per group:
     within  = 0.3110                                         min =         23
     between = 0.6994                                         avg =       50.3
     overall = 0.3314                                         max =         63

                                                F(4,8866)         =    1000.68
corr(u_i, Xb)  = 0.0550                         Prob > F          =     0.0000

------------------------------------------------------------------------------
lrgdpna_growth |     Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+--------------------------------------------------------------
lrgdpna_growth |
           L1. |  .1540919   .0089917    17.14   0.000      .136466    .1717177
               |
  lccon_growth |  .3630015   .0068018    53.37   0.000     .3496684    .3763346
    lck_growth |  .0272539   .0096247     2.83   0.005     .0083872    .0461206
   lpop_growth |  .4357088   .0539432     8.08   0.000     .3299676      .54145
         _cons |  .7646936   .1235722     6.19   0.000     .5224634    1.006924
---------------+--------------------------------------------------------------
       sigma_u |  .9448029
       sigma_e | 5.3589959
           rho | .03014548   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(179, 8866) = 1.23                  Prob > F = 0.0206
```

A Stata example: Anderson and Hsiao (1982) IV approach

- `xtivreg lrgdpnagrowth (l.lrgdpnagrowth=l2.lrgdpnagrowth) lccongrowth lckgrowth lpopgrowth, fe`

```
Fixed-effects (within) IV regression            Number of obs      =     8,871
Group variable: country1                        Number of groups   =       180

R-sq:                                           Obs per group:
     within  = 0.2947                                         min =        22
     between = 0.6076                                         avg =      49.3
     overall = 0.3128                                         max =        62

                                                Wald chi2(4)       =   8126.16
corr(u_i, Xb)  = 0.0421                         Prob > chi2        =    0.0000

------------------------------------------------------------------------------
lrgdpna_growth |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------+--------------------------------------------------------------
lrgdpna_growth |
           L1. |   .1484253   .0433245     3.43   0.001     .0635108    .2333399
               |
  lccon_growth |   .3441003   .0074722    46.05   0.000     .3294551    .3587455
    lck_growth |   .0286314   .0115522     2.48   0.013     .0059894    .0512733
   lpop_growth |   .4464321    .063721     7.01   0.000     .3215412     .571323
         _cons |   .8671765   .1323043     6.55   0.000     .6078648    1.126488
---------------+--------------------------------------------------------------
       sigma_u |  1.0085497
       sigma_e |   5.269231
           rho |  .03534062   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F  test that all u_i=0:     F(179,8687) =     0.86         Prob > F    = 0.9180
```

Topic 4: The GMM approach

- As we mentioned in last lecture, we can use IV and GMM approach to deal with the endogeneity.
- We have studied the endogenous problem and how to use IV approach to deal with it.
- In this lecture, we will review/study the GMM approach. To begin, we first review the GMM estimation in the classical linear regression model. Then, we will extend to the dynamic panel model!

Review: Moment condition for linear regression models

- Consider the classical linear regression model with exogenous regressors as studied in Lecture 1:

$$\boldsymbol{y} = \boldsymbol{x}\beta + \mu.$$

- Therefore, the following $k \times 1$ moment condition holds

$$E(x_i \mu_t) = E(x_i(y_i - x_i^\top \beta)) = 0.$$

- Note that the above equations form a set of what we may call theoretical moment. Each theoretical moment condition corresponds to a sample moment, or empirical moment, of the form conditions

$$g_n(\beta) = \frac{1}{n} \sum_{i=1}^{n} x_i(y_i - x_i^\top \beta).$$

Review: The general setup of the MM and GMM

- Before, we only assume and use the moment condition $E(x_i\mu_t) = E(x_i(y_i - x_i^\top\beta)) = 0$. In fact, this can be greatly relaxed. Now, suppose economic theory or common sense provide a set of m moment conditions and $m \geq k$: $E(g_i(\beta)) = 0_{m\times 1}$.
- Let $g_n(\beta)$ be the sample analogue of $E(g_i(\beta))$.
- Example: By using a valid set of instruments z_i, we can construct $m \times 1$ moment condition where $g_i(\beta) = z_i(y_i - x_i^\top\beta)$ and the sample analogue is $g_n(\beta) = \frac{1}{n}\sum_{i=1}^{n}(z_i(y_i - x_i^\top\beta))$.
- MM (Method of Moments): The MM estimator solves the following m equations

$$g_n(\widehat{\beta}^{MM}) = 0.$$

- GMM (Generalized Method of Moments): Given a $m \times m$ weighting matrix W, the GMM estimator solves the optimization problem

$$\widehat{\beta}^{GMM} = arg\min_{\beta\in\Theta_\beta} Q_n(\beta)^{GMM} = arg\min_{\beta\in\Theta_\beta} g_n(\beta)^\top W g_n(\beta).$$

Review: MM estimator for classic linear regression model

- Consider we use $E(x_i \mu_i) = 0$. The sample counterpart equations are

$$\frac{1}{n}\sum_{i=1}^{n} x_i (y_i - x_i^\top \beta)$$

- Therefore, MM estimator gives

$$\frac{1}{n}\sum_{i=1}^{n} x_i (y_i - x_i^\top \beta) = 0$$

$$\Longrightarrow \widehat{\beta}^{MM} = (\sum_{i=1}^{n} x_i x_i^\top)^{-1} (\sum_{i=1}^{n} x_i y_i)$$

- Remark: The OLS estimator is a MM estimator.

Review: GMM estimator for classic linear regression model

- Again, consider we use $E(x_i\mu_i) = 0$. The GMM estimator solves

$$\begin{aligned}\min_{\beta\in\Theta_\beta} Q_n(\beta)^{GMM} &= g_n(\beta)^\top W g_n(\beta)\\ &= \left(\frac{1}{n}\boldsymbol{x}^\top(\boldsymbol{y}-\boldsymbol{x}\beta)\right)^\top W\left(\frac{1}{n}\boldsymbol{x}^\top(\boldsymbol{y}-\boldsymbol{x}\beta)\right).\end{aligned}$$

- The FOC gives

$$\begin{aligned}&\boldsymbol{x}^\top\boldsymbol{x}W\boldsymbol{x}^\top(\boldsymbol{y}-\boldsymbol{x}\widehat{\beta}^{GMM}) = 0\\ \Longrightarrow\ &\widehat{\beta}^{GMM} = \left(\boldsymbol{x}^\top\boldsymbol{x}W\boldsymbol{x}^\top\boldsymbol{x}\right)^{-1}\boldsymbol{x}^\top\boldsymbol{x}W\boldsymbol{x}^\top\boldsymbol{y}.\end{aligned}$$

- Remark: If $W = I_{m\times m}$, $\widehat{\beta}^{GMM} = (\boldsymbol{x}^\top\boldsymbol{x})^{-1}\boldsymbol{x}^\top\boldsymbol{y}$. The GMM estimator becomes OLS estimator.
- At home: Try to solve the GMM estimator with moment condition $E(z_i\mu_i) = 0$.

Review: Asymptotic results for GMM estimator

- Theorem: Under some regularity conditions (Newey and Mcfadden 1994), we can show

$$\sqrt{n}\left(\widehat{\beta}^{GMM}-\beta_0\right)\xrightarrow{d}N\left(0,(G^\top WG)^{-1}G^\top W\Omega WG(G^\top WG)^{-1}\right),$$

- $G=G(\beta_0)$, $G(\beta)=\frac{\partial E[g(\beta)]}{\partial\beta}$, and $\Omega=E\left[g(\beta)g(\beta)^\top\right]$

Proof.

Let $G_n(\beta)$ be the derivative of $g_n(\beta)$. By applying the *Lagrange mean value theorem*, the FOC gives

$$\frac{\partial Q_n(\beta)^{GMM}}{\partial\beta}=2G_n(\widehat{\beta}^{GMM})^\top W\left[g_n(\beta_0)+G_n(\bar{\beta})\left(\widehat{\beta}^{GMM}-\beta_0\right)\right]=0$$

$$\Longrightarrow\sqrt{n}\left(\widehat{\beta}^{GMM}-\beta_0\right)=-\left[G_n(\widehat{\beta}^{GMM})^\top WG_n(\bar{\beta})\right]^{-1}G_n(\widehat{\beta}^{GMM})^\top W\sqrt{n}g_n(\beta_0)$$

$$=-\left[G^\top WG\right]^{-1}G^\top W\sqrt{n}g_n(\beta_0)+o_p(1).$$

Then assuming i.i.d. and applying Lindberg-Levy CLT to $\sqrt{n}g_n(\beta_0)$ yields $\sqrt{n}g_n(\beta_0)\xrightarrow{d}N(0,\Omega)$, which completes the proof. ∎

Review: Feasible efficient GMM estimator

- The asymptotic result implies that the optimal weighting matrix is $W = \Omega^{-1}$ and gives the most efficient GMM estimator with

$$\sqrt{n}\left(\widehat{\beta}^{GMM} - \beta_0\right) \xrightarrow{d} N\left(0, (G^\top W G)^{-1}\right).$$

- Hence, a two-step feasible efficient GMM is proposed as follows:
  1. Step 1: Using $W = I_{m\times m}$ and obtain a pre-estimator $\widehat{\beta}$ and compute

$$\widehat{\Omega} = \frac{1}{n-1}\sum_{i=1}^{n} g(\widehat{\beta})g(\widehat{\beta})^\top.$$

  2. Step 2: Using $W = \widehat{\Omega}$ and obtain $\widehat{\beta}^{GMM}$.

Review: Remarks on two-step GMM

- Remark 1: Empirical results show that the bias of the above feasible efficient GMM estimator is significant when the sample size is small, see the special issue of the Journal of Business and Economic Statistics 1996.

- Remark 2: This problem can be alleviated by the continuous-updating GMM estimator proposed by Hansen, Heaton, and Yaron (Hansen *et al.* 1996). The intuition is simple: Repeat the above two-steps for many more times.

Topic 4: GMM approach to dynamic panel data models

- Now, we turn back to a dynamic panel AR(1) model.

$$y_{it} = \gamma y_{i,t-1} + \alpha_i + \varepsilon_{it}.$$

- Various GMM estimators (i.e. moment conditions) have been proposed for dynamic panel data models
  1. Arellano and Bond (1991): GMM estimator
  2. Arellano and Bover (1995): GMM estimator
  3. Ahn and Schmidt (1995): GMM estimator
  4. Blundell and Bond (1998): a system GMM estimator
- Key insights: They propose to use different moment conditions.

Topic 4: Arellano and Bond (1991) estimator

- Consider the first-difference dynamic panel AR(1) model

$$\Delta y_{it} = \gamma \Delta y_{i,t-1} + \Delta \varepsilon_{it}.$$

- Arellano and Bond (1991) propose to use the following moment conditions

$$E\left[\Delta \varepsilon_{it} y_{it-j}\right] = 0,$$
$$\text{For } j = 2, \ldots, (t-1).\ \ t = 3, \ldots, T.$$

- Therefore, (Arellano and Bond 1991) have $m = (T-2)(T-1)/2$ linear moment restrictions.

Topic 4: Arellano and Bover (1995) estimator

- The first-difference transform has a weakness. It magnifies gaps in unbalanced panels - If If some y_{it} is missing, for example, then both Δy_{it} and Δy_{it+1} are missing.
- This motivates the second common transformation, called "forward orthogonal deviations" or "orthogonal deviations" (Arellano and Bover 1995).
- Intuition of forward orthogonal deviations: subtracting the average of all future available observations of a variable.
- A set of moment conditions can be constructed by using the forward transformed data.

Topic 4: Ahn and Schmidt (1995) estimator

- Consider again the first-difference dynamic panel AR(1) model

$$
\begin{aligned}
y_{it} &= \gamma y_{i,t-1} + u_{it}, \\
u_{it} &= \alpha_i + \varepsilon_{it}.
\end{aligned}
$$

- Under regular assumptions, (Ahn and Schmidt 1995) find the following holds:
  - For all i, ε_{it} is uncorrelated with y_{i0} for all t.
  - For all i, ε_{it} is uncorrelated with α_i for all t
  - For all i, ε_{it} are nutually uncoorelated.

- Hence, besides the previous ones, they propose the following moment condition holds:
  1. $E(y_{it}\Delta u_{it+1} - y_{it+1})\Delta u_{it+2}) = 0$ for $t = 1, \ldots, T-2$.
  2. $E(\bar{u}_i \Delta u_{it+1}) = 0$ for $t = 1, \ldots, T-1$.
  3. $E(u_{iT}\Delta y_{it}) = 0$ for all $t = 1, \ldots, T-1$.
  4. $E(u_{it}y_{it} - u_{it-1}y_{it-1}) = 0$ for all $t = 2, \ldots, T$.

Topic 4: Blundell and Bond (1998) system GMM estimator

- As underscored in (Blundell and Bond 1998), Arellano and Bond instruments may be weak if the dynamic process is persistent. For example, if $T = 3$, the model we considered at $t = 2$ becomes

$$y_{i2} = \gamma y_{i1} + \alpha_i + \varepsilon_{i2}.$$

- By deducting both sides by y_{i1}, we have

$$\Delta y_{i2} = (\beta - 1) y_{i1} + \alpha_i + \varepsilon_{i2}.$$

- Key observation: As $\beta \longrightarrow 1$, the correlation between Δy_{i2} and y_{i1} decreases. Arellano and Bond istruments are weak in this sense.

- To solve the weak instrument problem, Blundell and Bond (1998) propose to add the following additional moment conditions

$$E\left[\Delta y_{it-1}(\alpha_i + \varepsilon_{it})\right] = 0$$

Reference

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