

ECON 3740: INTRODUCTION TO ECONOMETRICS

INSTRUCTOR: CHAOYI CHEN
Department of Economics and Finance, University of Guelph

Lecture 13

Last lecture, we studied the standardized OLS estimator, under several necessary assumptions, follows t distribution. Today, we will

- Construct t statistic
- Hypothesis test - one sided
- Hypothesis test - two sided
- Compute P value for t test
- The confidence interval

MLR: t Statistic

- Suppose the **null hypothesis** is

$$H_0 : \beta_j = 0$$

- The population parameter, under the null, is equal to zero, i.e. after controlling for the other independent variables, there is no effect of x_j on y .
- The **t statistic** or **t ratio** of $\hat{\beta}_j$ for above hypothesis test is

$$t_{\hat{\beta}_j} = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)}$$

- The t statistic will be used to test the above null hypothesis. The farther the estimated coefficient is away from zero, the less likely it is that the null hypothesis holds true. But what does "far" away from zero mean?
- This depends on the variability of the estimated coefficient, i.e., its standard deviation. The t statistic measures how many estimated standard deviations the estimated coefficient is away from zero.
- Under the null,

$$t_{\hat{\beta}_j} = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)} \sim t_{n-k-1}$$

MLR: Testing against One-Sided Alternatives

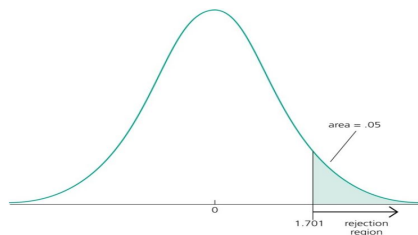
- **Goal:** Define a rejection rule so that, if H_0 is true, it is rejected only with a small probability (= **significance level** or level for short, e.g., 5%).
- To determine the rejection rule, we need to decide the relevant **alternative hypothesis**.
- First consider a **one-sided alternative** of the form

$$H_1 : \beta_j > 0$$

- Reject the null hypothesis in favor of the alternative hypothesis if the estimated coefficient is "too large" (i.e. larger than a **critical value**).
- Construct the critical value so that, if the null hypothesis is true, it is rejected in, for example, 5% of the cases.

MLR: Testing against One-Sided Alternatives

Figure: 5% Rejection Rule for the Alternative $H_1 : \beta_j > 0$ with 28 df



- In the figure, this is the point of the t -distribution with 28 degrees of freedom that is exceeded in 5% of the cases.
- So the rejection rule is to reject if the t statistic is greater than 1.701.
- **Analog:** evidences cannot happen if you are not a criminal in testing H_0 : you are not a criminal vs. H_1 : you are a criminal.

MLR: Example: Hourly Wage Equation

- Test whether, after controlling for education and tenure, higher work experience leads to higher hourly wages.

- The fitted regression line is

$$\widehat{\log(wage)} = 0.284 + 0.092educ + 0.0041exper + 0.022tenure$$

(0.104) (0.007) (0.0017) (0.003)

where $n = 526$, $R^2 = 0.316$, and standard errors appear in parentheses below the estimated coefficients.

- Test

$$H_0 : \beta_{exper} = 0 \text{ \& } H_1 : \beta_{exper} > 0$$

- One would either expect a positive effect of experience on hourly wage or no effect at all.

MLR: Example: Hourly Wage Equation Continue

- The t statistic for $\hat{\beta}_{exper}$ is

$$t_{exper} = \frac{0.0041}{0.0017} \approx 2.41$$

- $df = n - k - 1 = 526 - 3 - 1 = 522$ in this example, which is quite large (standard normal approximation applies.).
- Check with z table (you can also check one sided t table), the 5% critical value is $c_{0.05} = 1.645$, and the 1% critical value is $c_{0.01} = 2.326$. Note that 10%, 5%, and 1% are conventional significance levels.
- The null hypothesis is rejected because the t statistic exceeds the critical value.
- The conclusion is that the effect of experience on hourly wage is statistically greater than zero at the 5% (and even at the 1%) significance level.

MLR: Testing against One-Sided Alternatives (less than zero) - an example

- Similar to alternative greater than zero, we can Testing against One-Sided Alternatives (less than zero).
- Example: Student Performance and School Size. Test whether smaller school size leads to better student performance.
- The fitted regression line is

$$\widehat{math10} = 2.274 + 0.00046totcomp + 0.048staff - 0.0002enroll$$

$(6.113) \quad (0.0001) \quad (0.040) \quad (0.00022)$

where $n = 408$, $R^2 = 0.0541$, and

$math10$ = percentage of students passing 10th-grade math test

$totcomp$ = average annual teacher compensation

$staff$ = staff per one thousand students

$enroll$ = school enrollment (= school size)

MLR: Testing against One-Sided Alternatives (less than zero) - an example continue

- Test

$$H_0 : \beta_{enroll} = 0 \ \& \ H_1 : \beta_{enroll} < 0$$

- Do larger schools hamper student performance or is there no such effect?

- The t statistic for $\hat{\beta}_{enroll}$ is

$$t_{enroll} = \frac{-0.0002}{0.00022} \approx -0.91$$

- $df = n - k - 1 = 408 - 3 - 1 = 404$, which is large, so the standard normal approximation applies.
- The 5% critical value is $c_{0.05} = -1.65$.
- The null hypothesis is not rejected because the t statistic is not smaller than the critical value.
- The conclusion is that one cannot reject the hypothesis that there is no effect of school size on student performance

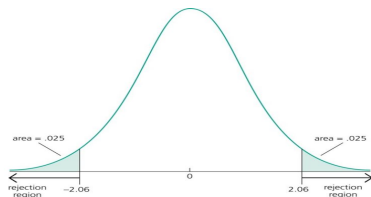
MLR: Testing against Two-Sided Alternatives

- We want to test

$$H_0 : \beta_j = 0 \text{ \& } H_1 : \beta_j \neq 0$$

- Reject the null hypothesis in favor of the alternative hypothesis if the **absolute** value of the estimated coefficient is too large.
- Construct the critical value so that, if the null hypothesis is true, it is rejected in, for example, 5% of the cases.

Figure: 5% Rejection Rule for the Alternative $H_1 : \beta_j \neq 0$ with 25 df



- In the figure, these are the points of the t-distribution so that 5% of the area is in the two tails.
- So the rejection rule is to reject if the t statistic is greater than 2.06 or less than -2.06.

MLR: Testing against Two-Sided Alternatives - an example

- The fitted regression line is

$$\widehat{colGPA} = 1.39 + 0.412hsGPA + 0.015ACT - 0.083skipped$$

(0.33) (0.094) (0.011) (0.026)

where $n = 141$, $R^2 = 234$, and

$skipped$ = average number of lectures missed per week

- Test

$$H_0 : \beta_{skipped} = 0 \text{ \& } H_1 : \beta_{skipped} \neq 0$$

- Is the number of lectures missed per week bad for college GPA?

- The t statistic for $\widehat{\beta}_{skipped}$ is

$$t_{enroll} = \frac{-0.083}{0.026} \approx -3.19$$

- $df = n - k - 1 = 141 - 3 - 1 = 137$. Hence, The 1% critical value is $c_{0.01} = 2.58$. (Note that this is a **two-sided** test).
- $t_{enroll} = 3.19 > c_{0.01}$. Therefore, the null hypothesis is rejected and the effect of $skipped$ is significantly different from zero at the 1% significance level.

MLR: Guidelines for Discussing Economic and Statistical Significance

- If a regression coefficient is different from zero in a two-sided test, the corresponding variable is said to be **statistically significant**.
- If a variable is statistically significant, discuss the magnitude of the coefficient to get an idea of its **economic** or **practical** significance.
- The fact that a coefficient is statistically significant does not necessarily mean it is economically or practically significant!
- If a variable is statistically and economically important but has the "wrong" sign, the regression model might be **misspecified**.
- If a variable is statistically insignificant at the usual levels (10%, 5%, 1%), one may think of dropping it from the regression.
- However, if the sample size is small, effects might be imprecisely estimated so that the case for dropping insignificant variables is less strong.

MLR: Computing p -Values for t Tests

- Evidently, if the significance level is made smaller and smaller, there will be a point where the null hypothesis cannot be rejected anymore.
- The reason is that, by lowering the significance level, one wants to avoid more and more to make the error of rejecting a correct H_0 .
- The smallest significance level at which the null hypothesis is still rejected, is called the p -value of the hypothesis test.
 - The p -value is the significance level at which one is indifferent between rejecting and not rejecting the null hypothesis.
 - Alternatively, the p -value is the probability of observing a t statistic as extreme as we did if the null is true.
 - A null hypothesis is rejected if and only if the corresponding p -value is smaller than the significance level.
- A small p -value is evidence against the null hypothesis because one would reject the null hypothesis even at small significance levels.
- A large p -value is evidence in favor of the null hypothesis.
- P -values are more informative than tests at fixed significance levels because you can choose your own significance level.

MLR: Computing p -Values for t Tests

- Recall that $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t_{n-k-1}$. Hence,

$$P\left(\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} > c_{0.05}\right) = 0.025$$

$$P\left(\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} < -c_{0.05}\right) = 0.025$$

$$P(c_{0.025} \leq \frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \leq c_{0.025}) = 0.95$$

where $c_{0.05}$ is the 5% critical value of two-sided test

- Simple manipulation of above result shows

$$\begin{aligned} P\left(\underbrace{\hat{\beta}_j - c_{0.05} * se(\hat{\beta}_j)}_{\text{lower bound of the CI}} \leq \beta_j \leq \underbrace{\hat{\beta}_j + c_{0.05} * se(\hat{\beta}_j)}_{\text{upper bound of the CI}}\right) \\ = P\left(\left|\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)}\right| \leq c_{0.05}\right) = 0.95 \end{aligned}$$

- The confidence interval is $[\hat{\beta}_j - c_{0.05} * se(\hat{\beta}_j), \hat{\beta}_j + c_{0.05} * se(\hat{\beta}_j)]$. 0.95 is called the **confidence level**.

MLR: Confidence Interval - an example

- The fitted regression line is

$$\widehat{\log(rd)} = \begin{matrix} -4.38 \\ (0.47) \end{matrix} + \begin{matrix} 1.084\log(\text{sales}) \\ (0.06) \end{matrix} + \begin{matrix} 0.0217\text{profmarg} \\ (0.0128) \end{matrix} \quad \text{where}$$

rd = firms spending on RD

sales = annual sales

profmarg = profits as percentage of sales

- $df = n - k - 1 = 32 - 2 - 1 = 29$. Hence, $c_{0.05} = 2.045$.
- The 95% CI for $\beta_{\log(\text{sales})}$ is $[1.084 - 2.045 * (0.06), 1.084 + 2.045 * (0.06)] = [0.961, 1.21]$. The effect of $\log(\text{sales})$ on $\log(rd)$ is relatively precisely estimated as the interval is narrow. Moreover, the effect is significantly different from zero because zero is outside the interval.
- The 95% CI for β_{profmarg} is $[0.0217 - 2.045 * 0.0128, 0.0217 + 2.045 * 0.0128] = [-0.0045, 0.0479]$. The effect of profmarg on $\log(rd)$ is imprecisely estimated as the interval is very wide. It is not even statistically significant because zero lies in the interval.