Empirical Panel Data: Lecture 7

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Topic 3: Panel data model estimators

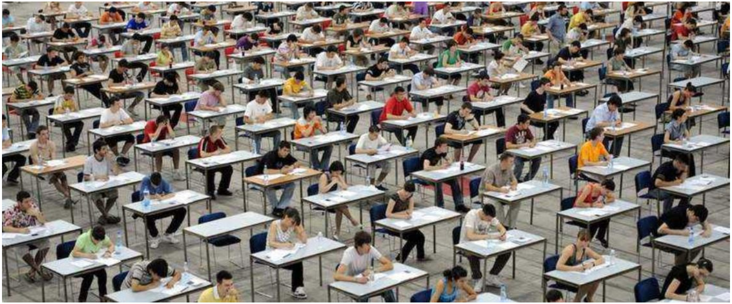
- Fixed effects model: LSDV estimator
- Random effects model: GLS estimator
- When T is large: **No** difference in whether to treat the effects as fixed or random. This is because both the LSDV estimator and the generalized least-squares estimator become the same estimator:

$$\widehat{\beta}^{GLS} \underset{T \longrightarrow \infty}{\longrightarrow} \widehat{\beta}^{LSDV}$$

- When T is finite: It can make a surprising amount of difference in the estimates of the parameters. We need to identify the effects. **We need to have a specification test!**

Topic 3: Specification testing?

Of course, not this test .....



Topic 3: Hypothesis testing

- Two fundamental assumptions in random effects:
  1. the unobserved individual effects α_i are random draws from a common population.
  2. The explanatory variables are strictly exogenous:

$$E(\varepsilon_{it}|\boldsymbol{X_i}) = E(\alpha_i|\boldsymbol{X_i}) = 0$$

- Therefore, we are interested in the following hypothesis testing:

$$H_0: \ E(\alpha_i|\boldsymbol{X_i}) = 0$$
$$H_A: \ E(\alpha_i|\boldsymbol{X_i}) \neq 0$$

- Under the null, the model is a random effect model.
- Under the alternative, the model is a fixed effect model.

Topic 3: Hausman's specification test

- How to test? Hausman's approach.
- Hausman (1978) proposes a general specification test, that can be applied in the specific context of linear panel models to the issue of specification of individual effects (fixed or random).

source: Department of Economics, MIT

Topic 3: General idea of the Hausman's test

$$H_0: \ E(\alpha_i|\boldsymbol{X_i}) = 0$$
$$H_A: \ E(\alpha_i|\boldsymbol{X_i}) \neq 0$$

- Our observations:
  1. Under the null, true model is a random effects model. Both $\widehat{\beta}^{GLS}$ and $\widehat{\beta}^{LSDV}$ are consistent estimator. However, only $\widehat{\beta}^{GLS}$ is BLUE.
  2. Under the alternative, true model is a fixed effects model. $\widehat{\beta}^{GLS}$ is inconsistent. $\widehat{\beta}^{LSDV}$ is still consistent.
- Hausman's idea: examining the distance between $\widehat{\beta}^{GLS}$ and $\widehat{\beta}^{LSDV}$
  1. If the distance is small, H_0 can not be rejected.
  2. If the distance is large, H_0 can be rejected.

Topic 3: Distance measure and the Wald statistic

- Naturally, we can use the *standardized* distance defined as follows:

$$H = \left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)^\top \left(\text{Var}\left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)\right)^{-1} \left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)$$

- **Remark 1**: This is a typical *Wald* statistic. Under the null, $H \xrightarrow{d} \chi^2(k)$.
- **Remark 2**: However, this test statistic is not feasible to use since we cannot compute it!
- **What we know?** $\widehat{\beta}^{LSDV}$, $\widehat{\beta}^{GLS}$, $\text{Var}(\widehat{\beta}^{LSDV})$, $\text{Var}(\widehat{\beta}^{GLS})$
- **What we do not know?** $\text{Var}\left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)$.
- Why? Because we do not know $Cov(\widehat{\beta}^{GLS}, \widehat{\beta}^{LSDV})$.

Topic 3: Hausman's Lemma

- Under the null, Hausman (Hausman 1978) shows

$$\text{Var}\left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right) = \boxed{\text{Var}(\widehat{\beta}^{LSDV}) - \text{Var}(\widehat{\beta}^{GLS})}.$$

- Therefore, Hausman suggests using the test statistic

$$H = \left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)^{\top} \left(\text{Var}(\widehat{\beta}^{LSDV}) - \text{Var}(\widehat{\beta}^{GLS})\right)^{-1} \left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)$$

- KEY-INSIGHT of Hausman's Lemma:
  - Under the null, the difference between $\widehat{\beta}^{GLS}$ and $\widehat{\beta}^{LSDV}$ is orthogonal to $\widehat{\beta}^{GLS}$. Thus,

$$Cov(\widehat{\beta}^{GLS}, \widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}) = 0 \iff Cov(\widehat{\beta}^{GLS}, \widehat{\beta}^{LSDV}) = \text{Var}(\widehat{\beta}^{GLS})$$
$$\implies \text{Var}\left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right) = \text{Var}(\widehat{\beta}^{LSDV}) - \text{Var}(\widehat{\beta}^{GLS})$$

Topic 3: Hausman specification test asymptotic theory

- The Hausman specification test statistic of individual effect can be defined as follows:

$$H = \left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)^{\top} \left(\mathrm{Var}(\widehat{\beta}^{LSDV}) - \mathrm{Var}(\widehat{\beta}^{GLS})\right)^{-1} \left(\widehat{\beta}^{GLS} - \widehat{\beta}^{LSDV}\right)$$

- Theorem: Under $H_0: \ E(\alpha_i|\boldsymbol{X_i}) = 0$, we have

$$H \underset{nT \longrightarrow \infty}{\overset{d}{\longrightarrow}} \chi^2(k)$$

Topic 3: Remarks on Hausman specification test

- Hausman test applies to the typical case in practice When n is large relative to T.
- When n is fixed and T tends to infinity, $nT \longrightarrow \infty$ still holds. However, in this situation the fixed-effects and random-effects models become indistinguishable for all practical purposes. Moreover, the numerator and denominator of H approach zero at the same speed. see e.g. (Ahn and Moon 2014).

A Stata example: Hausman specification test

- `xtreg lrgdpnagrowth lccongrowth lckgrowth lpopgrowth, fe`
- `estimates store fixed`
- `xtreg lrgdpnagrowth lccongrowth lckgrowth lpopgrowth, re`
- `estimates store random`
- `hausman fixed random, sigmamore`

```
. hausman fixed random, sigmamore

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     fixed        random       Difference          S.E.
-------------+----------------------------------------------------------------
lccon_growth |    .3785448     .3837601      -.0052153        .0007537
  lck_growth |    .0511562     .0704687      -.0193125         .002678
 lpop_growth |     .566651     .4287039        .137947        .0382441
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       95.47
                Prob>chi2 =      0.0000
```

Reference

Ahn, Seung Chan and Hyungsik Roger Moon (2014). Large-n and large-t properties of panel data estimators and the hausman test. *Festschrift in Honor of Peter Schmidt* p. 219–258.

Hausman, J. A. (1978). Specification tests in econometrics. *Econometrica* **46**(6), 1251–1271.