Threshold MIDAS Forecasting of Inflation Rate*

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Abstract

We propose several threshold mixed data sampling (TMIDAS) autoregressive models to forecast the Canadian inflation rate using predictors observed at different frequencies. These models take two low-frequency variables and a high-frequency index as a threshold variable. We compare our TMIDAS models to commonly used benchmark models, evaluating their in-sample and out-of-sample forecasts. Our results demonstrate the good forecasting performance of the TMIDAS models. Particularly, the in-sample results highlight that the TMIDAS model using the high-frequency index as the threshold variable outperforms other models. Through unconditional superior predictive ability (USPA) and conditional superior predictive ability (CSPA) tests for out-of-sample evaluation, we find that no single model consistently outperforms the others, although at least one of our TMIDAS models remains competitive in most cases.

JEL Classification: C24; C53
Keywords: Forecasting; High-frequency index; Mixed data sampling; Superiority predictive ability test; Threshold regression.

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1 Introduction

Understanding the movement of a country's economic activity is crucial for policy decision-making process. Hence, the timely assessment of the movements of key macroeconomics variables such as inflation is important to guide policymakers in formulating appropriate policies as well as to mitigate the impact of a shock. Inflation is one of the variables that both central banks and market practitioners continuously monitor. With the objectives to promote price stability, central banks routinely monitor inflation expectations and forecasts, whereas market practitioners aim to modify their investment strategies by exploiting any new information that is released. However, it is accepted in the economics literature that inflation is hard to forecast (see, e.g., Stock & Watson (2008)), and accurately forecasting inflation has become a major challenge for the government and central banks.

There has been a recent surge of interest in the academic literature to forecast inflation, including, among others, Cecchetti et al. (2000), Stock & Watson (2003), Banerjee & Marcellino (2006), Stock & Watson (1999, 2010, 2016), Paap & Ravazzolo (2013), and Chen et al. (2014), where a range of models are used. In addition to the traditional long-horizon predictions, more attention has recently been paid to short-horizon predictions, or commonly known as now-casting, which takes advantage of the real-time availability and the mixed frequency nature of the data. It is believed that higher-frequency data carry valuable information and are necessary in producing accurate short-horizon forecasts. For example, by adopting a dynamic factor model, Modugno (2013) suggested models that only incorporating monthly data perform worse than models using mixed frequency data. Andreou et al. (2013) showed that quarterly forecasts of GDP in the U.S improve when daily data is included in a mixed data sampling (MIDAS) model compared to the forecasts from models that only use low-frequency variables. The MIDAS method, which was developed by Ghysels and his co-authors in the late 2000s (see e.g, Ghysels et al. (2004), Ghysels et al. (2005) and Ghysels et al. (2007)) provides an econometric solution to forecast economic variables recorded at low frequency in a parsimonious way using time series data recorded at both low and high frequencies.

To the best of our knowledge, no previous studies in the literature have explored the presence of threshold effects within a mixed frequency framework, with the exception of Yang (2021). Typically, the prevailing approach assumes a linear relationship between the dependent variable and its predictors, disregarding substantial empirical evidence that supports the existence of nonlinear relationships, including threshold effects. Notably, threshold effects have been examined in various contexts, such as US GNP estimation and the impact of public debt on economic output, as discussed in studies by Caner et al. (2010), Cecchetti et al. (2011), Afonso & Jalles (2013), Chudik et al. (2017), Lee et al. (2017)), among others. Threshold modeling has found widespread applications in economics and finance, as highlighted by Hansen (2011), who provided an overview of its use in forecasting variables such as output growth, interest rates, prices, exchange rates and stock returns. Building upon this motivation, our paper introduces several threshold models within a mixed-frequency framework to examine the predictability of high-frequency predictors for the Canadian inflation rate.

In the literature on inflation forecasting, researchers have explored various predictors to identify reliable and useful indicators. For instance, Cologni & Manera (2008) demonstrated the influence of oil price changes on inflation, while Chen et al. (2014) showed that world commodity price aggregates can predict CPI and PPI inflation, particularly when accounting for structural breaks. Groen et al. (2013) reassessed the accuracy of inflation forecasting by incorporating quarterly activity and expectations variables which include lagged values of inflation, a host of real activity data, term structure data, (relative) price data and surveys. Among them, the real food commodity price index and the real raw material commodity price index have been considered. In fact, the commodity price index is widely regarded as a reliable indicator (see, e.g., Stock & Watson (1999), Browne & Cronin (2010), Gospodinov & Ng (2013) and Modugno (2013)). Moreover, financial indicators have been found to be valuable in real-time inflation tracking, benefiting policy-making. For example, Forni et al. (2003) employ financial variables such as interest rates, yield spreads, and exchange rates from a large panel of time series data to forecast inflation. Stock & Watson (2003) analyzed quarterly data for seven OECD countries and identify key predictors of inflation, including stock price indices and exchange rates. In our analysis, we incorporate financial variables such as the Toronto exchange price index, USD-CAD exchange rate together with the commodity price index. Additionally, considering the divergence in the literature regarding the choice and empirical findings related to short-term interest rates versus yield spreads as predictors (Stock & Watson (2003)), we include Canada's three-month bond yield as another candidate predictor. All these variables are based on high-frequency weekly data in our analysis.

In summary, our paper makes several contributions. Firstly, while there has been growing interest in forecasting or now-casting GDP using mixed frequency data frameworks (see, e.g., Giannone et al. (2008), Angelini et al. (2011), and Bańbura & Rünstler (2011)), there is relatively less work on forecasting inflation within this context. Exceptions include studies such as Breitung & Roling (2015), Monteforte & Moretti (2013) and more recently, Gorgi et al. (2019). Breitung & Roling (2015) imposed a non-parametric lag distribution function to the standard MIDAS model, while Monteforte & Moretti (2013) adopted a generalized dynamic factor model in the mixed frequency model. Gorgi et al. (2019) proposed a mixed-frequency score-driven dynamic model that transforms the predictive log-likelihood score contributions of the high-frequency variables through a MIDAS weighting scheme. In our paper, we extend this mixed-frequency approach and introduce a threshold effect to the MIDAS model, resulting in a more general TMIDAS model. This approach allows for a more realistic and flexible nonlinear relationship between the dependent variable and predictors. Furthermore, we explore different threshold variables within our TMIDAS model to enhance forecast accuracy. Unlike Yang (2021), which only used low-frequency threshold variables, we include a high-frequency index as a threshold variable in our model. Our results demonstrate that incorporating the high-frequency index yields the best performance for the TMIDAS model in the in-sample analysis, aligning with expectations that higher-frequency data contain valuable additional information worth leveraging.

The remainder of the paper is organized as follows. Section 2 introduces our proposed threshold MIDAS models. Section 3 describes the estimation procedure for the proposed models. In Section 4, we report both the in-sample and out-of-sample forecasting performance of our proposed models and benchmark models including autoregressive, augmented autoregressive, autoregressive distributed lag (ADL), MIDAS, and threshold effect models. A robustness check is also discussed in this section. Finally, Section 5 concludes the paper.

2 Threshold MIDAS Models

A simple h-step-ahead forecasting MIDAS model is defined as

$$y_{t+h} = \beta_0 + \beta_1 B\left(L^{1/m};\theta\right) x_t^{(m)} + \varepsilon_{t+h} \tag{2.1}$$

for $t = 1, \ldots, T$, where y_{t+h} is a low-frequency variable to be forecast from a high-frequency variable $x_t^{(m)}$ which is observed between time $t-1$ and t. And, $B\left(L^{1/m};\theta\right) = \sum_{j=0}^{m-1} B\left(j;\theta\right) L^{j/m}$ is a polynomial function of the lag operator, $L^{1/m}$, of the high frequency variable up to degree $m-1$, where $L^{j/m} x_t^{(m)} = x_{t-j/m}^{(m)}$, j is the number of high-frequency lags in the temporal aggregation of $x_{t/m}^{(m)}$, and the coefficients $B(j;\theta)$ are known up to a finite number of parameters θ in Ghysels et al. (2007). Therefore, we can regard $B\left(L^{1/m};\theta\right)x_t^{(m)}$ as a low-frequency variable constructed from a weighted average of high-frequency data, and name this term as the *MIDAS* term for ease of reference. Clements & Galvão (2008) and Andreou et al. (2013) extend the above model to an autoregressive MIDAS($d+1$,$q+1$) model as

$$y_{t+h} = \alpha_0 + \sum_{l=0}^{d} \alpha_l y_{t-l} + \sum_{l=0}^{q} \beta_l B\left(L^{1/m};\theta\right) x_{t-l}^{(m)} + \varepsilon_{t+h}, \tag{2.2}$$

where both the lagged variables of the low-frequency variable back to time $t-d$ and the high-frequency variables back to $t-q$ are used to make a forecast of y_{t+h} at time t.

Furthermore, there have been other extensions for MIDAS models in more recent studies. Foroni et al. (2015) recommended an unrestricted MIDAS model without imposing any structure on $B(j;\theta)$ when m is small. Building upon the unrestricted MIDAS model of Foroni et al. (2015), Siliverstovs (2017) extended the targeted predictor method of Bai & Ng (2008) to predict the quarterly real GDP growth rate in Switzerland from a large group of monthly data. Breitung & Roling (2015) further relaxed the parametric lag distribution function to a non-parametric lag distribution function with certain smoothness property and propose to estimate the unknown lag distribution function via a penalized least-squares estimator.

However, from an economic point of view, the aforementioned MIDAS model essentially assumes the predicting ability of high-frequency variables is constant, while in reality, it can change across regimes, following, for example, a certain market condition or a business cycle. To this end, there are other models that have been developed to introduce non-linearity to MIDAS models considered in Ghysels et al. (2007). For example, Becker & Osborn (2012) applied the weighted smooth transition regression approach to introduce non-linearity where the smoothed transition function is an exponential function of a linear combination of high-frequency variables; Guérin & Marcellino (2013) introduced a Markov switching MIDAS model that incorporates regime changes in the parameters of the MIDAS model. In this paper, we propose an alternative way to allow for the non-linearity by applying the threshold regression modeling of Tong & Lim (1980) to the MIDAS approach, i.e., a threshold MIDAS model. As explained in Tong & Lim (1980) and Tong (2011), the general formulation of the threshold model includes the Markov switching model, hence our threshold MIDAS has a more general setting.

Note that we can rewrite model (2.2) as

$$y_{t+h} = \boldsymbol{\alpha}^\top z_t + \boldsymbol{\beta}^\top \mathbf{x}_t(\theta) + \varepsilon_{t+h}, \tag{2.3}$$

where $\boldsymbol{\alpha} = [\alpha_0, \ldots, \alpha_d]^\top$, $z_t = [1, y_t, y_{t-1}, \ldots, y_{t-d}]^\top$, $\boldsymbol{\beta} = [\beta_0, \ldots, \beta_q]^\top$, $\mathbf{x}_t(\theta) = [x_t(\theta), \ldots, x_{t-q}(\theta)]^\top$, and $x_{t-l}(\theta) = B\left(L^{1/m}; \theta\right) x_{t-l}^{(m)}$ for $l = 0, \ldots, q$.

Firstly, incorporating the threshold regression approach to model (2.3), we propose a threshold mixed data sampling (TMIDAS) model:

$$y_{t+h} = \begin{cases} \boldsymbol{\alpha}_1^\top z_t + \boldsymbol{\beta}_1^\top \mathbf{x}_t(\theta_1) + \varepsilon_{t+h}, \ w_t \leq \gamma_0, \\ \boldsymbol{\alpha}_2^\top z_t + \boldsymbol{\beta}_2^\top \mathbf{x}_t(\theta_2) + \varepsilon_{t+h}, \ w_t > \gamma_0, \end{cases} \tag{2.4}$$

where w_t is a low-frequency threshold variable used to split the sample into two groups, and γ_0 is the true threshold parameter. Note that model (2.4) allows the weighting parameters, θ_1 and θ_2, to be different in two regimes. Hence, our model offers more flexibility than the TMIDAS model proposed by Yang (2021), which strictly assumes the weighting parameter is invariant across the two regimes.

As it is possible that more than one economic factor can result in regime switches of future inflation rates, we extend model (2.4) further by allowing for the threshold to depend on a linear index of multiple economic factors. Introducing a threshold index to a threshold regression is considered in Seo & Linton (2007) but for time series data observed at equal frequency. Specifically, we propose our second TMIDAS model with a threshold index as follows:

$$y_{t+h} = \begin{cases} \boldsymbol{\alpha}_1^\top z_t + \boldsymbol{\beta}_1^\top \mathbf{x}_t(\theta_1) + \varepsilon_{t+h}, \ \mathbf{w}_t^\top \boldsymbol{\gamma}_1 \leq \gamma_0, \\ \boldsymbol{\alpha}_2^\top z_t + \boldsymbol{\beta}_2^\top \mathbf{x}_t(\theta_2) + \varepsilon_{t+h}, \ \mathbf{w}_t^\top \boldsymbol{\gamma}_1 > \gamma_0, \end{cases} \tag{2.5}$$

where $\mathbf{w}_t = [w_{1t}, \ldots, w_{qt}]^\top$, a vector of low frequency threshold variables, and $\boldsymbol{\gamma}_1 = [\gamma_1, \ldots, \gamma_q]^\top$.

Both model (2.4) and model (2.5) split the sample based on low frequency economic factor(s), using either a single variable or a linear index of more than one variable. Since in many empirical applications, there is no reason to expect a discontinuity that must depend on the value of a low-frequency variable, as a modification, we propose the following *high-frequency-TMIDAS* model, where the future path is determined by a high-frequency index instead. Hence, our third TMIDAS model is given by

$$y_{t+h} = \begin{cases} \boldsymbol{\alpha}_1^\top z_t + \boldsymbol{\beta}_1^\top \mathbf{x}_t(\theta_1) + \varepsilon_{t+h}, \ hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) \leq \gamma_0, \\ \boldsymbol{\alpha}_2^\top z_t + \boldsymbol{\beta}_2^\top \mathbf{x}_t(\theta_2) + \varepsilon_{t+h}, \ hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) > \gamma_0, \end{cases} \tag{2.6}$$

where $hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) = \mathbf{x}_t(\tilde{\theta})^\top \boldsymbol{\gamma}_1 = \sum_{l=0}^{q} \gamma_l x_{t-l}(\tilde{\theta})$, $\boldsymbol{\gamma}_1 = [\gamma_{10}, \gamma_{11}, \ldots, \gamma_{1q}]^\top$, and we normalize $\gamma_{10} = 1$. We want to point our that the selection of a high-frequency predictor, $x_t^{(m)}$, has a dual impact on model (2.6). It not only affects the *MIDAS* terms, $\{x_{t-l}(\cdot)\}_{l=0}^{q}$, but also determines the construction of the threshold variable, hfi_t. Note that, given $\tilde{\theta}$, we can regard $x_{t-l}(\tilde{\theta})$ as a low frequency variable that is constructed as a weighted average of high frequency variables given weight $\tilde{\theta}$, and in that regards, model (2.6) essentially returns to model (2.5), where $\mathbf{w}_t = [x_t(\tilde{\theta}), x_{t-1}(\tilde{\theta}), \ldots, x_{t-q}(\tilde{\theta})]^\top$.

In model (2.6), we consider a scalar high frequency variable $x_t^{(m)}$. In practice, more than one high-frequency variable may be useful. For example, we observe $k(> 1)$ high-frequency variables and let $\mathbf{f}_t^{(m)} = \left[f_{t-(m-1)/m}^{(m)}, \ldots, f_{t-j/m,}^{(m)}, \ldots, f_{t-1/m}^{(m)}, f_t^{(m)}\right]$ be the $k \times m$ matrix observed in period t. Following Lee et al. (2021), we apply the principal component analysis (PCA) method to obtain both the latent high-frequency threshold factor and the high-frequency predictor. Specifically, let $\mathbf{f}^{(m)} = \left[\mathbf{f}_1^{(m)}, \ldots, \mathbf{f}_2^{(m)}, \ldots, \mathbf{f}_T^{(m)}\right]^T$ be the $(mT) \times k$ high-frequency data matrix. We calculate the first principle component of $\mathbf{f}^{(m)}$ to obtain $\mathbf{x}^m = \left[\mathbf{x}_1^{(m)}, \mathbf{x}_2^{(m)}, \ldots, \mathbf{x}_T^{(m)}\right]^T$, where $\mathbf{x}_t^{(m)} = \left[x_{t-(m-1)/m}^{(m)}, \ldots, x_{t-j/m,}^{(m)}, \ldots, x_{t-1/m}^{(m)}, x_t^{(m)}\right]$. In this way, we construct a high-frequency predictor, named as *PCA*.

3 Estimation

Models (2.4)-(2.6) are three TMIDAS predictive regression models. In this section, we describe the estimation procedure for model (2.6), and model (2.4) can be estimated by taking similar but easier steps. Note that we can re-write model (2.6) as

$$y_{t+h} = \boldsymbol{\delta}_1^\top \boldsymbol{\chi}_t(\theta_1) I\left(hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) \leq \gamma_0\right) + \boldsymbol{\delta}_2^\top \boldsymbol{\chi}_t(\theta_2) I\left(hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) > \gamma_0\right) + \varepsilon_{t+h}, \tag{3.1}$$

where $\boldsymbol{\delta}_1 = \left[\boldsymbol{\alpha}_1^\top, \boldsymbol{\beta}_1^\top\right]^\top$, $\boldsymbol{\delta}_2 = \left[\boldsymbol{\alpha}_2^\top, \boldsymbol{\beta}_2^\top\right]^\top$, $\boldsymbol{\chi}_t(\theta_j) = \left[z_t^\top, \mathbf{x}_t(\theta_j)^\top\right]^\top$ for $j = 1, 2$, and $I(\cdot)$ is the indicator function.

Following the literature (e.g., Ghysels et al. (2007), Hansen (2000), and Yang (2021)), we apply the least squares (LS) estimator to estimate the model. Letting $\theta = [\theta_1^\top, \theta_2^\top]^\top$, we construct the LS objective function as

$$SSR_T(\boldsymbol{\delta}_1, \boldsymbol{\delta}_{2,}, \theta, \tilde{\theta}, \boldsymbol{\gamma}_1, \gamma_0) = \sum_{t=q+1}^{T-h} \left[y_{t+h} - \boldsymbol{\delta}_1^\top \boldsymbol{\chi}_t(\theta_1) I\left(hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) \leq \gamma_0\right) - \boldsymbol{\delta}_2^\top \boldsymbol{\chi}_t(\theta_2) I\left(hfi_t(\tilde{\theta}, \boldsymbol{\gamma}_1) > \gamma_0\right)\right]^2.$$

Let $\boldsymbol{\gamma} = [\gamma_0, \boldsymbol{\gamma}_1^\top]^\top$ be the threshold parameter vector. We assume $\tilde{\theta} \in \Theta_{\tilde{\theta}}$, $[\gamma_0, \boldsymbol{\gamma}_1^\top]^\top \in \Gamma^{q+2}$, and $(\boldsymbol{\delta}_1, \boldsymbol{\delta}_{2,}, \theta) \in \boldsymbol{\Theta}$, where $\Theta_{\tilde{\theta}}$, Γ, and $\boldsymbol{\Theta}$ are all compact sets. Given $(\tilde{\theta}, \boldsymbol{\gamma})$, the LS estimator is given by

$$\left(\widehat{\boldsymbol{\delta}}_1(\tilde{\theta}, \boldsymbol{\gamma}), \widehat{\boldsymbol{\delta}}_2(\tilde{\theta}, \boldsymbol{\gamma}), \widehat{\theta}(\tilde{\theta}, \boldsymbol{\gamma})\right) = \underset{(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \theta) \in \boldsymbol{\Theta}}{\arg\min} \; SSR_T(\boldsymbol{\delta}_1, \boldsymbol{\delta}_{2,}, \theta, \tilde{\theta}, \boldsymbol{\gamma}). \tag{3.2}$$

Then, substituting the LS estimator of $(\boldsymbol{\delta}_1, \boldsymbol{\delta}_{2,}, \theta)$ as a function of $(\tilde{\theta}, \boldsymbol{\gamma})$ back to the objective function, given $\tilde{\theta}$, we have

$$\left(\widehat{\gamma}_0(\tilde{\theta}), \widehat{\boldsymbol{\gamma}}(\tilde{\theta})\right) = \underset{\boldsymbol{\gamma} \in \Gamma^{q+2}}{\arg\min} \; SSR_T\left(\widehat{\boldsymbol{\delta}}_1\left(\tilde{\theta}, \boldsymbol{\gamma}\right), \widehat{\boldsymbol{\delta}}_{2,}\left(\tilde{\theta}, \boldsymbol{\gamma}\right), \widehat{\theta}(\tilde{\theta}, \boldsymbol{\gamma}), \tilde{\theta}, \boldsymbol{\gamma}\right). \tag{3.3}$$

Therefore, the objective function is only about the weighting parameters of the high-frequency index, $\tilde{\theta}$, and we can estimate it by[1]

$$\widehat{\tilde{\theta}} = \underset{\tilde{\theta} \in \Theta_{\tilde{\theta}}}{\arg\min} \; SSR_T\left(\widehat{\boldsymbol{\delta}}_1\left(\tilde{\theta}, \boldsymbol{\gamma}(\tilde{\theta})\right), \widehat{\boldsymbol{\delta}}_2\left(\tilde{\theta}, \boldsymbol{\gamma}(\tilde{\theta})\right), \widehat{\theta}\left(\tilde{\theta}, \boldsymbol{\gamma}(\tilde{\theta})\right), \widehat{\boldsymbol{\gamma}}(\tilde{\theta}), \tilde{\theta}\right).$$

The threshold parameters are estimated as $\widehat{\boldsymbol{\gamma}} = \widehat{\boldsymbol{\gamma}}(\widehat{\tilde{\theta}})$ following a grid-search method which is widely used in the threshold literature. Other parameters are obtained as

$$(\widehat{\boldsymbol{\delta}}_1, \widehat{\boldsymbol{\delta}}_2, \widehat{\theta}) = \left(\widehat{\boldsymbol{\delta}}_1\left(\widehat{\tilde{\theta}}, \widehat{\boldsymbol{\gamma}}(\widehat{\tilde{\theta}})\right), \widehat{\boldsymbol{\delta}}_2\left(\widehat{\tilde{\theta}}, \widehat{\boldsymbol{\gamma}}(\widehat{\tilde{\theta}})\right), \widehat{\theta}\left(\widehat{\tilde{\theta}}, \widehat{\boldsymbol{\gamma}}(\widehat{\tilde{\theta}})\right)\right).$$

For the weighting function $B(j;\eta)$, following Ghysels et al. (2007), the most popularly used are two-parameter exponential Almon lag polynomial specification

$$B(j;\eta) = \frac{e^{\eta_1 j + \eta_2 j^2}}{\sum_{j=1}^{m} e^{\eta_1 j + \eta_2 j^2}},$$

or the two-parameter Beta Lag specification

$$B(j;\eta) = \frac{f(\frac{j}{m}, \eta_1; \eta_2)}{\sum_{j=1}^{m} f(\frac{j}{m}, \eta_1; \eta_2)},$$

where

$$f(x, a; b) = \frac{x^{a-1}(1-x)^{b-1}\Lambda(a+b)}{\Lambda(a)\Lambda(b)},$$
$$\Lambda(a) = \int_0^\infty e^{-x} x^{a-1} dx.$$

We use the two-parameter Beta function in our study.

[1] In practice, $\Theta_{\tilde{\theta}}$ can be chosen by the practitioner as a pre-specified grid. Our study uses a 20% trimming rate to construct the grid search interval.

4 Inflation Forecasting Using TMIDAS Models

During the 1970s, when using data with the same sampling frequency, the commodity price index was considered as a leading indicator for forecasting inflation in inflation forecasting models. However, the significant correlation between commodity prices and inflation weakened by the 1980s. Similarly, there is mixed evidence for the correlation between financial indicators and inflation over time. These shifts in predictability motivate us to extend the MIDAS approach to a broader setting, enabling the predictive ability of high-frequency predictors on low-frequency inflation to vary based on observed high or low-frequency economic factors. In this paper, we employ mixed frequency data, i.e., monthly inflation rate and weekly predictors as we describe in Section 4.1, to re-investigate such correlation, with a focus on short-term relationships. Among others, we incorporate the potential threshold effect in our predictive regression by applying model (2.4) and our proposed model (2.6).[2] In contrast to most of the studies in the literature that mainly look at the US or European countries, our paper focuses on the Canadian inflation forecast.

4.1 Data

The data used in our study were obtained from the Bank of Canada's website and Yahoo Finance. There are two types of data we use. One type of data is the monthly low-frequency data from 1992:M6 to 2021:M1, which includes the Consumer Price Index (CPI) and core CPI. The annualized month-over-month total inflation rate is defined as $100((CPI_{t+1}/CPI_t)^{12} - 1)$, where CPI_t represents the monthly CPI index in period t for $t = 1, 2, ..., T$. This is the dependent variable studied in our paper. Similar to the total inflation rate, we define the annualized month-over-month core inflation rate based on the core CPI. The difference between the total and core inflation rates is referred to as the annualized month-over-month inflation difference, which we term "Diff inflation" in our analysis. Both core inflation and Diff inflation are considered threshold variables in our study. This choice is motivated by the significance of the total inflation rate for consumers and the core inflation rate for the Bank of Canada's monetary policy. The core inflation rate is designed to capture the long-term trend of total inflation, while the rate differentials capture short-term

[2] In this study, we are more interested in examining the linear (or nonlinear) predictive power of high-frequency data on the inflation rate. Therefore, we focus on model (2.4) and model (2.6).

deviations from the Bank of Canada's operational inflation target.[3]

The other type of data is the weekly high-frequency data from the first week of June 1992 to the last week of January 2021, which includes the percentage change of Bank of Canada's commodity price index (BCPI), Toronto stock exchange price index (TSX), USD-CAD exchange rate (ER), and Canada's three-month bond yield (3M-yield). We construct PCA, the first principle component of the observed four weekly variables as explained in the last paragraph of Section 2. In our analysis, $x_t^{(4)}$ is referred to as either BCPI, TSX, ER, 3M-yield, or PCA.[4] Table 1 summarizes the descriptive statistics of all these variables.

Table 1: **Descriptive Statistics**

Variable	Mean	Max.	Min.	Std. Dev.
Total inflation	1.9308	14.7639	-11.7600	4.1280
Core inflation	1.8608	10.5541	-6.8796	3.1290
Diff inflation	0.0701	8.9450	-16.5270	3.1033
BCPI	0.0914	23.4982	-14.3211	1.9388
TSX	0.1182	13.6744	-16.0894	2.2146
ER	0.0114	8.3600	-5.1000	1.0646
3M-yield	-0.1060	189.4737	-50.2024	7.3439
PCA	0.0000	24.3760	-8.7869	1.2982

Figures 1-5 show time series plots of the monthly dependent variable against all the five high-frequency variables. The visual inspection of the plots indicates that both individual variables and the common factor can capture the pattern of the dependent variable to some extent, supporting our choice of these explanatory variables in our analysis.

4.2 In-sample results

We make the one-month-ahead ($h = 1$) and three-month-ahead ($h = 3$) total inflation rate forecasts from several competitive models and the in-sample estimation results are reported in Tables 2-6, where each table contains two panels, Panel A and Panel B, for the one-month-ahead and three-month-ahead forecasts, respectively. To investigate the forecasting

[3]The former governor of the Bank of Canada, Mr. Mark Carney, reiterated the Bank's monetary policy at the Board of Trade of Metropolitan Montreal in Montreal, Quebec on November 23, 2011: " ...The Bank continues to use core inflation as an operational guide for its monetary policy because it is an effective indicator of the underlying trend in CPI inflation. Core inflation, along with other measures of inflationary pressures, is monitored to help achieve the target for total CPI inflation; it is not a replacement for the latter."

[4]Here, m=4 as there are 4 weekly observations per month.

Figure 1: Monthly Total Inflation Rate and BCPI Changes

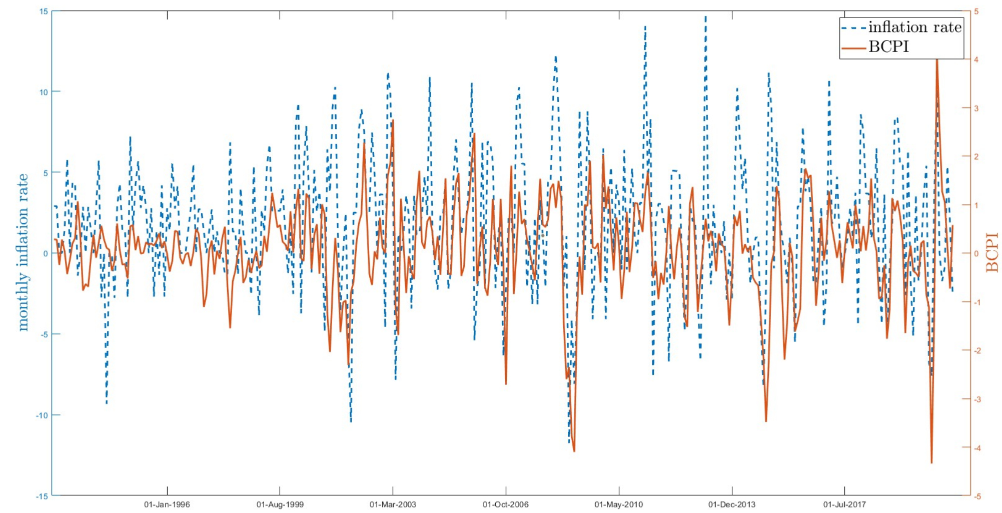
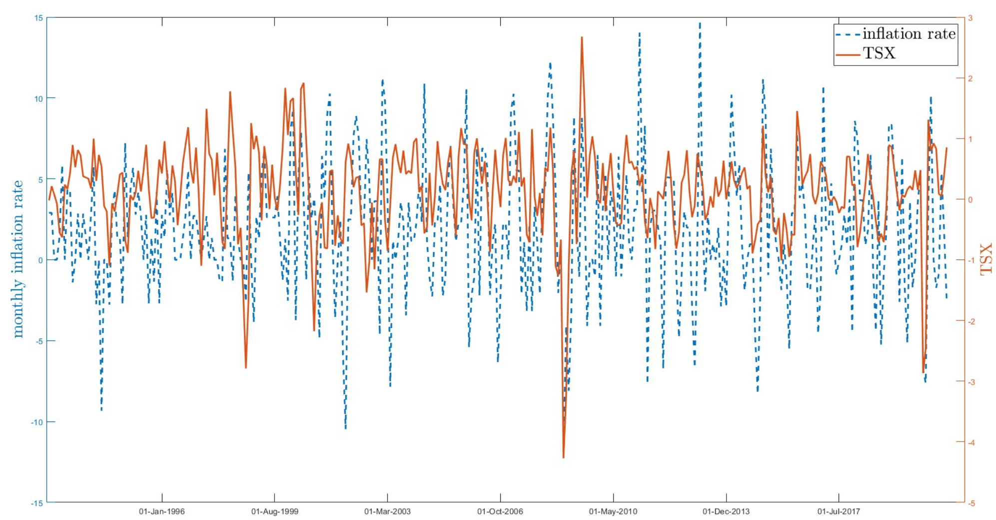


Figure 2: Monthly Total Inflation Rate and TSX Returns



performance of the TMIDAS models (2.4) and (2.6), we compare it with other five competing models in Tables 2-6. We give detailed definitions of each model below.

The first model is a simple AR(1) model of total inflation rate listed in the second column, which nests to model (2.2) with $d = 0$ and $\boldsymbol{\beta} = 0$. This model is used to evaluate how well the historical values of total inflation rate can be used to forecast the future total inflation

Figure 3: Monthly Total Inflation Rate and ER Changes

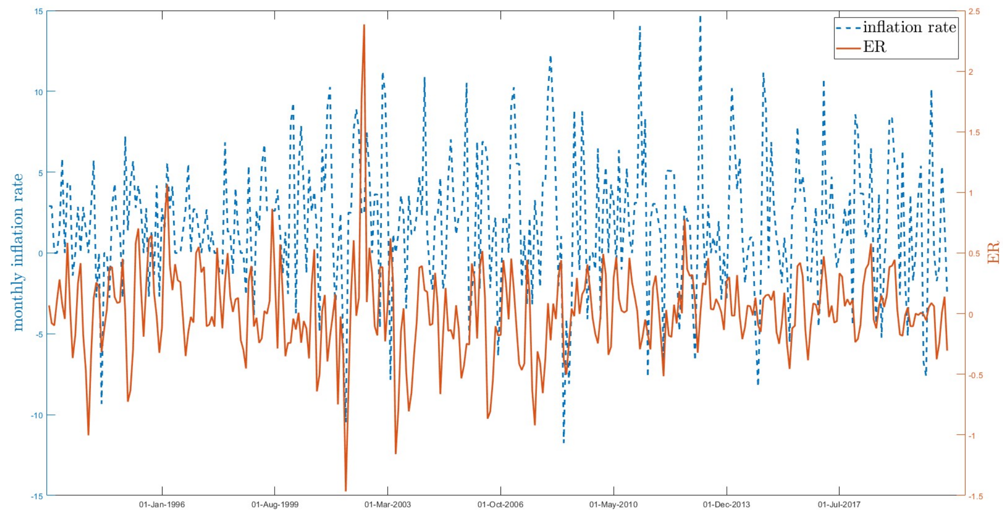
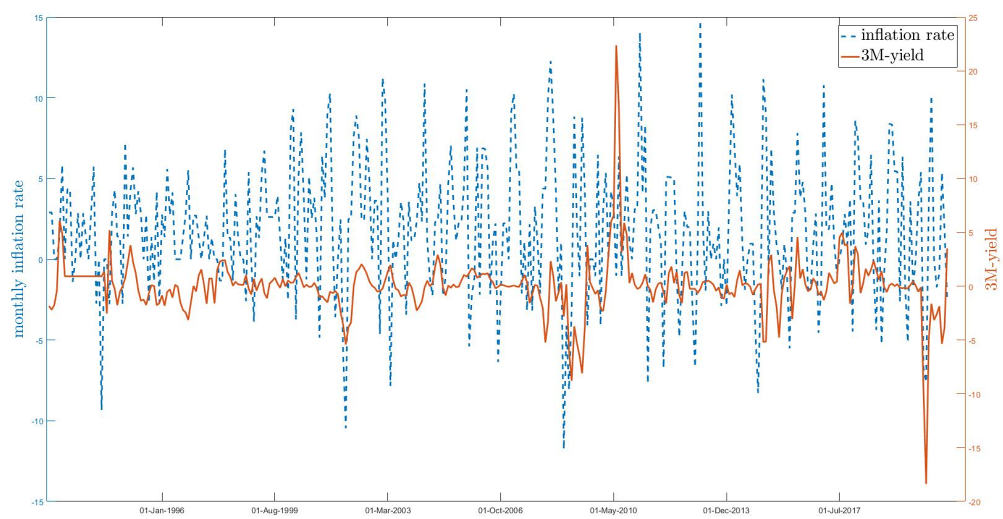
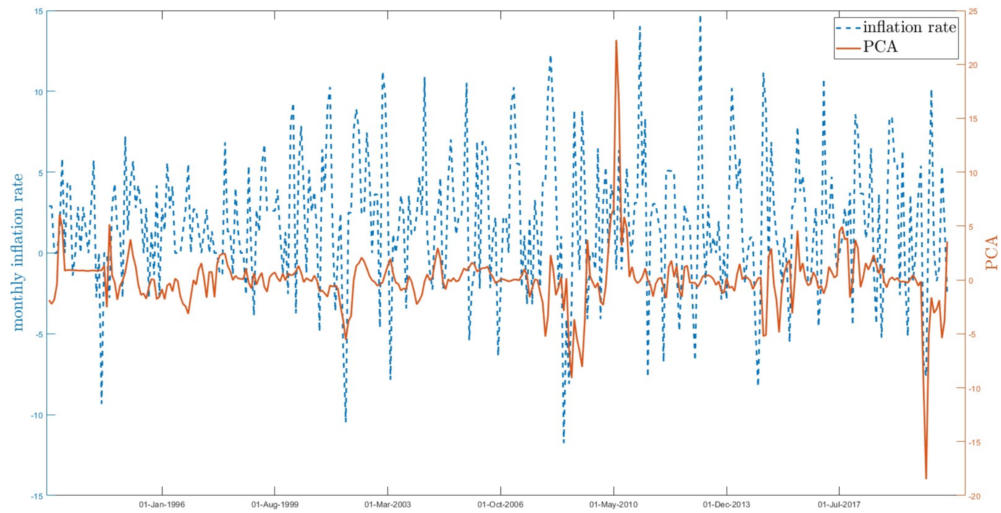


Figure 4: Monthly Total Inflation Rate and 3M-yield Changes



rates. In Column 3, our second candidate model augments the AR(1) model with one of the five high-frequency variables, where the high-frequency predictor ($x_t^{(4)}$) in Tables 2-6 are BCPI, TSX, ER, 3M-yield, and PCA, respectively. For example, suppose that we are in month t, and we use BCPI as the high-frequency variable to predict the total inflation rate in month $t+h$. Then the augmented AR(1) model uses the total inflation rate and all four

Figure 5: Monthly Total Inflation Rate and *PCA*



BCPI observations in month t to forecast the total inflation rate in month $t+h$.

We notice that the second model contains a total of six parameters after including the lagged weekly observed variables.[5] We then consider two more models to parsimoniously use the high-frequency variables in Columns 4 and 5. Specifically, the third candidate model reported in Column 4 uses the weekly average of one of the high-frequency variables in month t, which reduces the number of parameters to 3. We call it the ADL(1,1) model that utilizes the total inflation rate and weekly average of the high-frequency variable in month t as the predictor of total inflation in month $t+h$, where the high-frequency variable used in Tables 2-6 is the same as in the second model.[6] The fourth candidate model, reported in column 5, is the MIDAS model (2.2), where the $MIDAS$ term is used with its weights calculated from the $Beta$ function as explained in Section 3. The two models in columns 4 and 5 differ in terms of how we aggregate the weekly observations into monthly observations. Compared to the augmented AR(1) model, both $ADL(1,1)$ model and MIDAS model provide a parsimonious view of the trade-off between parameter proliferation and a much more extensive information set of high-frequency data.

[5]The model includes an intercept term, the total inflation rate and four weekly observations for one of the high-frequency variables in month t.

[6]The regression analysis of Augmented $AR(1)$ and $ADL(1,1)$ models includes lagged high-frequency variables but not reported to save space.

The block of columns beneath *Threshold* report the results from two threshold regression models, which exclude the high-frequency variables from model (2.4). Hence, the two models nest to model (2.4). And, one threshold model takes core inflation as the threshold variable, while the other takes inflation difference as the threshold variable.

The last block of columns beneath *TMIDAS* provide the estimate of our proposed TMIDAS models with three different threshold variable choices; that is, the TMIADS model defined by model (2.4) and the high-frequency-TMIDAS model defined by model (2.6), where the row name *MIDAS* denotes a low-frequency variable constructed from a weighted high-frequency variable as explained in Section 2. Specifically, columns *Core inflation* and *Diff inflation* refer to the TMIDAS models using the core inflation and inflation difference as the low-frequency threshold variable, respectively. Column *High-frequency index* refers to the TMIDAS model (2.6) in which the high-frequency index is constructed as the weighted average of a corresponding high-frequency predictor and is used to determine the sample splitting. Therefore, unlike the TMIDAS-Core and TMIDAS-Diff models, where the choice of the high-frequency predictor only affects the construction of the *MIDAS* term, selecting the high-frequency predictor also determines the threshold variable in the TMIDAS-HFI model. Again, the high-frequency predictor, $x_t^{(4)}$ in the TMIDAS models, in Tables 2-6, is the same as in the second to fourth candidate models. The parameters d and q in model (2.2) and model (2.6) are determined by the Akaike information criterion (AIC) and we use $d = 0$ and $q = 0$. By including *MIDAS* and *Threshold* as competing models, we are able to compare the forecasting performance with the TMIDAS model which contains both MIDAS and threshold effects in the model.

The in-sample results indicate the significance of the *MIDAS* term in our TMIDAS models in at least one regime[7], which confirms the importance of including a high-frequency predictor in explaining the low-frequency inflation rate.[8] Our proposed TMIDAS models outperform all other competing models in both one-step-ahead and three-step-ahead forecasts for all cases in Tables 2-6 in terms of both R^2 and the mean squared error (MSE) values. Additionally, it is evident that when the high-frequency index is used as the threshold variable, our TMIDAS model achieves the best overall performance confirmed by the largest R^2 and smallest MSE when the high-frequency predictor, BCPI, TSX, or *PCA*, is used for both one-month-ahead

[7]Except for one-month-ahead forecasts of TMIDAS-Diff.

[8]In addition to BCPI, TSX, ER, and 3M-yield, the findings are also supported by **PCA**, which is calculated as the first PCA of the four weekly variables, accounting for 42.343% of variation of the four high-frequency variables.

and three-month-ahead forecasting cases; and when 3M-yield is used as the predictor for the three-month-ahead forecasting case. Moreover, the TMIDAS model with the 3M-yield as the high-frequency predictor stands out with the highest R^2 and smallest MSE, which is seconded by the TMIDAS model with BCPI as the high-frequency predictor in the one-month-ahead case. Our results seem to be in favor of the view that 3M-yield is a useful predictor and provides strong predictive power for total inflation (see, e.g, Kozicki et al. (1997) and others). The predictive power of BCPI is consistent with the studies in the literature mentioned in Section 1 that the commodity price index is found to be a reliable indicator among other predictors (see, e.g., Stock & Watson (1999), Modugno (2013), Breitung & Roling (2015)).

4.3 Out-of-sample forecasts

In this section, we compare the out-of-sample (OOS) forecasting performance of the TMIDAS models (2.4) and (2.6) with other six competing models: Historical mean, traditional AR(1) model, the augmented AR(1) model, the ADL(1,1) model, the MIDAS model, and the threshold models. Data from June 1992 to December 2009 are used as the initial training period. We run rolling forecasting regressions and implement the *unconditional superior predictive ability (USPA)* test developed in White (2000) and then refined by Hansen (2005). Specifically, we consider the following null hypothesis:

$$H_0 : E\left[L_{t+h}\left(y_{t+h}, \hat{f}_{t,j}\right) - L_{t+h}\left(y_{t+h}, \hat{f}_{t,0}\right)\right] \geq 0, 1 \leq j \leq J,$$

where $L_{t+h}(\cdot)$ is the loss function defined as the sum of squared errors, $\hat{f}_{t,j}$ is the forecast estimate of y_{t+h} from model j, and the subscription 0 refers to the benchmark model. Therefore, the null hypothesis states that the benchmark is not inferior to any of the alternatives.

Tables 7-11 report the USPA test results at the 5% significance level. Under the column 'USPA', the one-versus-one test reports the number of rejections of a benchmark model in each row against each of the other nine competing methods. One-versus-all test reports the rejection of the benchmark in each row against all other nine competing methods, where we use symbol ✓ to indicate rejection.

Consider the one-versus-all test results first. In Table 11, using *PCA* as the high-frequency predictor, all our TMIDAS models show superior performance than non-TMIDAS competing models in both one-month-ahead and three-month-ahead forecasts. For models using other high-frequency predictors, at least one of our TMIDAS models exhibits better performance

Table 2: **In-Sample Forecasts Results with Predictor BCPI**

Panel A: One-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					4.1114		0.0025		1.0000		0.0025		-0.9773	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.5342***	1.5362***	1.5444***	1.5753***	1.7410***	-0.4270	0.9800***	2.4586***	1.3218***	1.3724***	2.1932***	1.3014***	2.0613***	-2.5244**
	(0.2442)	(0.2305)	(0.2404)	(0.2348)	(0.2551)	(0.8076)	(0.3153)	(0.4239)	(0.3820)	(0.3683)	(0.4167)	(0.3275)	(0.2754)	(1.2411)
y_t	0.1701***	0.1377**	0.1616***	0.4558***	0.1313*	0.4338***	0.1213	0.0734	0.4531***	0.8047***	0.3279*	0.6859**	0.2903***	-0.8626
	(0.0563)	(0.0599)	(0.0615)	(0.1455)	(0.0751)	(0.1210)	(0.0897)	(0.0863)	(0.1363)	(0.2267)	(0.1809)	(0.3164)	(0.0955)	(0.6045)
MIDAS				0.1147**					0.1942**	-0.0659	0.0679	0.0440	0.0727	0.3409**
				(0.0549)					(0.0776)	(0.1094)	(0.0771)	(0.0959)	(0.0570)	(0.1481)
R^2	0.0288	0.1146	0.0291	0.0874	0.0507		0.0457		0.1161		0.0999		0.1378	
MSE	16.7290	15.2513	16.7235	15.7188	16.3527		16.4377		15.2854		15.5036		14.8516	

Panel B: Three-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					0.9185		-1.9308		2.7860		2.3509		1.0233	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.7053***	1.7569***	1.7381***	1.7753***	2.5119***	1.0428***	1.2700**	1.5949***	0.6157	2.1152***	4.4743***	1.5900***	4.0888***	1.3868***
	(0.2931)	(0.2920)	(0.2874)	(0.2717)	(0.5117)	(0.4004)	(0.6099)	(0.3609)	(0.5990)	(0.3146)	(0.9780)	(0.2861)	(0.7756)	(0.3231)
y_t	0.1821***	0.1862***	0.1825***	0.2829	0.1993**	0.1283*	0.1704*	0.1859***	0.0873	0.4520**	-0.6246	0.3842**	-0.2683	0.1482
	(0.0575)	(0.0578)	(0.0572)	(0.1658)	(0.0970)	(0.0729)	(0.1010)	(0.0707)	(0.1609)	(0.2305)	(0.4083)	(0.1970)	(0.2316)	(0.1640)
MIDAS				0.1896***					0.1890**	0.1649**	0.0968	0.2039***	0.1608	0.1990***
				(0.0540)					(0.0888)	(0.0679)	(0.1239)	(0.0601)	(0.1078)	(0.0614)
R^2	0.0360	0.0520	0.0376	0.0479	0.0660		0.0468		0.0785		0.0839		0.0910	
MSE	16.6043	16.3299	16.5776	16.3997	16.0877		16.4198		15.8726		15.7795		15.6582	

Notes: $***$, $**$, and $*$ denote statistical significance at 1%, 5%, and 10% level, respectively. Standard errors are reported in parentheses.

Table 3: **In-Sample Forecasts Results with Predictor TSX**

Panel A: One-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					4.1114		0.0025		1.0000		1.2997		-0.7764	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.5342***	1.4780***	1.4902***	1.5043***	1.7410***	-0.4270	0.9800***	2.4586***	1.0224***	1.3724***	2.2023***	1.1923***	1.6856***	1.6980***
	(0.2442)	(0.2358)	(0.2365)	(0.2381)	(0.2551)	(0.8076)	(0.3153)	(0.4239)	(0.3745)	(0.3825)	(0.5939)	(0.2657)	(0.2711)	(0.5483)
y_t	0.1701***	0.1200**	0.1216**	0.7253**	0.1313*	0.4338***	0.1213	0.0734	0.6023**	1.3587***	1.0535***	1.2751***	0.3305*	1.2534***
	(0.0563)	(0.0571)	(0.0560)	(0.2845)	(0.0751)	(0.1210)	(0.0897)	(0.0863)	(0.2692)	(0.4052)	(0.3696)	(0.3921)	(0.1804)	(0.3411)
MIDAS				0.1283**					0.2379***	-0.0890	0.0932	0.0285	0.1678***	0.0021
				(0.0544)					(0.0755)	(0.1144)	(0.0962)	(0.0768)	(0.0618)	(0.0982)
R^2	0.0288	0.0758	0.0552	0.0603	0.0507		0.0457		0.0935		0.1002		0.1035	
MSE	16.7290	15.9196	16.2740	16.1871	16.3527		16.4377		15.6145		15.4991		15.4433	

Panel B: Three-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					0.9185		-1.9308		2.7940		2.2726		0.1665	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.7053***	1.6609***	1.7137***	1.6785***	2.5119***	1.0428**	1.2700**	1.5949***	0.5971	2.0495***	4.0232***	1.4335***	1.7726***	0.4715
	(0.2931)	(0.2836)	(0.2916)	(0.2684)	(0.5117)	(0.4004)	(0.6099)	(0.3609)	(0.5637)	(0.3166)	(0.9141)	(0.2773)	(0.4069)	(0.4937)
y_t	0.1821***	0.1839***	0.1801***	-0.1865	0.1993**	0.1283*	0.1704*	0.1859**	0.7437***	-0.1454	0.5630	-0.1969*	0.5131	-0.6073**
	(0.0575)	(0.0577)	(0.0571)	(0.0970)	(0.0970)	(0.0729)	(0.1010)	(0.0707)	(0.2537)	(0.1157)	(0.4128)	(0.1064)	(0.4494)	(0.2433)
MIDAS				0.1785***					0.2206**	0.1537**	0.1641	0.1882***	0.0971	0.2396***
				(0.0539)					(0.0873)	(0.0687)	(0.1117)	(0.0608)	(0.0729)	(0.0764)
R^2	0.0360	0.0721	0.0379	0.0463	0.0660		0.0468		0.0858		0.0846		0.1019	
MSE	16.6043	15.9824	16.5716	16.4274	16.0877		16.4198		15.747		15.7676		15.4696	

Notes: $***, **$, and $*$ denote statistical significance at 1%, 5%, and 10% level, respectively. Standard errors are reported in parentheses.

Table 4: **In-Sample Forecasts Results with predictor ER**

Panel A: One-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					4.1114		0.0025		1.2550		0.0448		-0.3177	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.5342***	1.5089***	1.5327***	1.4989***	1.7410***	-0.4270	0.9800***	2.4586***	1.0102***	1.7391***	2.5507***	1.0721***	1.8812***	0.8576**
	(0.2442)	(0.2437)	(0.2443)	(0.2425)	(0.2551)	(0.8076)	(0.3153)	(0.4239)	(0.4068)	(0.3428)	(0.4693)	(0.2940)	(0.3004)	(0.3982)
y_t	0.1701***	0.1619***	0.1649***	0.5738	0.1313*	0.4338***	0.1213	0.0734	0.5091	1.2708***	0.8220*	0.8324	0.4013	0.7589
	(0.0563)	(0.0565)	(0.0571)	(0.3805)	(0.0751)	(0.1210)	(0.0897)	(0.0863)	(0.4832)	(0.4560)	(0.4355)	(0.7378)	(0.3584)	(0.6476)
MIDAS				0.1639***					0.2729***	0.0646	0.0549	0.1180	0.0594	0.3691***
				(0.0532)					(0.0786)	(0.0955)	(0.0835)	(0.0878)	(0.0664)	(0.0894)
R^2	0.0288	0.0438	0.0304	0.0399	0.0507		0.0457		0.0860		0.0779		0.0765	
MSE	16.7290	16.4701	16.7008	16.5374	16.3527		16.4377		15.7429		15.8837		15.9077	

Panel B: Three-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					0.9185		-1.9308		2.7310		2.2726		0.2818	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.7053***	1.6692***	1.7026***	1.6823***	2.5119***	1.0428***	1.2700**	1.5949***	0.4924	2.0545***	4.1332***	1.4310***	2.7256***	1.1843***
	(0.2931)	(0.2934)	(0.2934)	(0.2702)	(0.5117)	(0.4004)	(0.6099)	(0.3609)	(0.5755)	(0.3209)	(0.9181)	(0.2808)	(0.6235)	(0.3363)
y_t	0.1821***	0.1841***	0.1821***	-0.2035	0.1993**	0.1283*	0.1704*	0.1859***	-1.0951	0.9621**	-0.7450	0.5498	0.4853	-0.5223*
	(0.0575)	(0.0576)	(0.0575)	(0.2086)	(0.0970)	(0.0729)	(0.1010)	(0.0707)	(0.7150)	(0.4354)	(0.7096)	(0.3459)	(0.5410)	(0.3149)
MIDAS				0.1816***					0.1524*	0.1589**	0.1706	0.2000***	0.0262	0.2449***
				(0.0541)					(0.0867)	(0.0695)	(0.1123)	(0.0616)	(0.1068)	(0.0622)
R^2	0.0360	0.0541	0.0363	0.0387	0.0660		0.0468		0.0816		0.0777		0.0757	
MSE	16.6043	16.2934	16.5995	16.5583	16.0877		16.4198		15.8189		15.8870		15.9215	

Notes: $***, **$, and $*$ denote statistical significance at 1%, 5%, and 10% level, respectively. Standard errors are reported in parentheses.

Table 5: **In-Sample Forecasts Results with Predictor 3M-yield**

Panel A: One-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					4.1114		0.0025		4.2970		0.9450		-0.8502	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.5342***	1.5411***	1.5356***	1.5154***	1.7410***	-0.4270	0.9800***	2.4586***	-0.4980	1.7519***	2.4373***	1.2067***	1.9049***	1.1445**
	(0.2442)	(0.2448)	(0.2428)	(0.2405)	(0.2551)	(0.8076)	(0.3153)	(0.4239)	(0.8463)	(0.2490)	(0.5559)	(0.2775)	(0.2928)	(0.5701)
y_t	0.1701***	0.1666***	0.1692***	0.0566	0.1313*	0.4338***	0.1213	0.0734	0.1091*	0.0750**	-0.1406*	0.1251	0.1099*	0.0685
	(0.0563)	(0.0557)	(0.0562)	(0.0349)	(0.0751)	(0.1210)	(0.0897)	(0.0863)	(0.0653)	(0.0364)	(0.0787)	(0.0934)	(0.0657)	(0.0665)
MIDAS				0.1698***					0.5004***	0.0781	0.1196	0.0908	0.1487**	0.1652*
				(0.0531)					(0.1262)	(0.0658)	(0.0987)	(0.0758)	(0.0651)	(0.0973)
R^2	0.0288	0.0509	0.0289	0.0389	0.0507		0.0457		0.2587		0.2537		0.2380	
MSE	16.7290	16.3481	16.7271	16.5557	16.3527		16.4377		12.7686		12.8552		13.1260	

Panel B: Three-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					0.9185		-1.9308		4.3980		0.9429		-0.1399	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.7053***	1.6642***	1.7183***	1.6483***	2.5119***	1.0428***	1.2700**	1.5949***	1.1939	1.7512***	2.7093***	1.2496***	1.4310***	1.4872***
	(0.2931)	(0.3068)	(0.2943)	(0.2789)	(0.5117)	(0.4004)	(0.6099)	(0.3609)	(1.0346)	(0.2964)	(0.6814)	(0.3325)	(0.3641)	(0.5059)
y_t	0.1821***	0.1897***	0.1770***	-0.1091**	0.1993**	0.1283*	0.1704*	0.1859***	-0.5310**	-0.1259*	-0.2290**	-0.0981	-0.0167	-0.1850
	(0.0575)	(0.0588)	(0.0579)	(0.0550)	(0.0970)	(0.0729)	(0.1010)	(0.0707)	(0.2171)	(0.0666)	(0.1048)	(0.0734)	(0.0240)	(0.1176)
MIDAS				0.1775***					0.1773	0.1737***	0.0897	0.1884***	-0.0356	0.3557***
				(0.0541)					(0.1451)	(0.0620)	(0.0971)	(0.0706)	(0.0743)	(0.0847)
R^2	0.0360	0.0574	0.0384	0.0494	0.0660		0.0468		0.2015		0.2091		0.2290	
MSE	16.6043	16.2365	16.5632	16.3746	16.0877		16.4198		13.7546		13.6225		13.2800	

Notes: $***$, $**$, and $*$ denote statistical significance at 1%, 5%, and 10% level, respectively. Standard errors are reported in parentheses.

Table 6: **In-Sample Forecasts Results with Predictor *PCA***

Panel A: One-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					4.1114		0.0025		1.0000		0.9840		0.3379	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.5342***	1.5421***	1.5408***	1.6001***	1.7410***	-0.4270	0.9800***	2.4586***	1.2278***	1.5837***	2.6097***	1.2455***	4.1647***	1.4640***
	(0.2442)	(0.2348)	(0.2402)	(0.2395)	(0.2551)	(0.8076)	(0.3153)	(0.4239)	(0.3912)	(0.3625)	(0.5475)	(0.2787)	(0.7725)	(0.3012)
y_t	0.1701***	0.1658***	0.1665***	0.8508***	0.1313*	0.4338***	0.1213	0.0734	0.8064*	1.5676***	1.2058*	1.0535*	-2.6838***	0.8715***
	(0.0563)	(0.0589)	(0.0592)	(0.2939)	(0.0751)	(0.1210)	(0.0897)	(0.0863)	(0.4192)	(0.4277)	(0.6333)	(0.5441)	(0.8696)	(0.2870)
MIDAS				0.1341**					0.2403***	-0.0286	0.0307	0.0900	0.2118**	0.1045
				(0.0554)					(0.0770)	(0.1064)	(0.0951)	(0.0794)	(0.0911)	(0.0683)
R^2	0.0288	0.0913	0.0289	0.0624	0.0507		0.0457		0.0911		0.0858		0.1053	
MSE	16.7290	15.7231	16.7270	16.2833	16.3527		16.4377		15.8319		15.8913		15.6146	

Panel B: Three-month-ahead

Model	*AR(1)*	*Augmented AR(1)*	*ADL*	*MIDAS*	*Threshold*				*TMIDAS*					
					Core Inflation		*Diff Inflation*		*Core Inflation*		*Diff Inflation*		*High Frequency Index*	
γ_0					0.9185		-1.9308		4.9680		2.1282		-0.3288	
Regime					Low	High	Low	High	Low	High	Low	High	Low	High
Constant	1.7053***	1.7187***	1.7006***	1.6922***	2.5119***	1.0428**	1.2700**	1.5949***	0.3872	1.7725***	3.6684***	1.4703***	1.3617***	1.9993***
	(0.2931)	(0.2924)	(0.2883)	(0.2709)	(0.5117)	(0.4004)	(0.6099)	(0.3609)	(1.1014)	(0.2803)	(0.9215)	(0.2786)	(0.3667)	(0.4435)
y_t	0.1821***	0.1756***	0.1818***	-0.1288	0.1993**	0.1283*	0.1704*	0.1859**	-2.8902**	0.5305	-2.5011**	-0.0845	0.6083	-1.0506***
	(0.0575)	(0.0587)	(0.0580)	(0.1992)	(0.0970)	(0.0729)	(0.1010)	(0.0707)	(1.1454)	(0.5109)	(1.1095)	(0.1690)	(0.5239)	(0.3373)
MIDAS				0.1786***					0.0809	0.1983***	0.1624	0.1950***	0.1858**	0.2520**
				(0.0544)					(0.1512)	(0.0575)	(0.1078)	(0.0621)	(0.0663)	(0.0905)
R^2	0.0360	0.0436	0.0361	0.0372	0.0660		0.0468		0.0641		0.0738		0.0845	
MSE	16.6043	16.4733	16.6034	16.5839	16.0877		16.4198		16.1216		15.9541		15.7688	

Notes: $***$, $**$, and $*$ denote statistical significance at a 1%, 5%, and 10% level, respectively. Standard errors are reported in parentheses.

than competing models. Particularly, using the 3M-yield or TSX as the high-frequency predictor, all of the three TMIDAS models outperform other competing models for three-month-ahead forecasts.

Next, to obtain a better understanding of the performance among all considered models, we look at the number of times that pairwise USPA tests reject the benchmark model in each row against each of the other nine competing models. How do our proposed TMIDAS models perform? It depends on the choice of the high-frequency predictor. If one uses the TMIDAS-HFI model to make one-month-ahead and three-month-ahead forecasts, our results will favor using TSX returns as the high-frequency predictor, using *PCA* for the TMIDAS-Core model, and using *PCA* and 3M-yield for the TMIDAS-Diff model. The results also indicate that joint USPA test results can be different from pairwise USPA test results.

4.4 Robustness check

A rejection or non-rejection of Hansen (2005)'s USPA test only suggests the models' forecasting performance *on average*. However, it does not exclude the possibility that a model can perform differently under certain given economic *states or conditions*. Thus, as a robustness check, we implement the *conditional* superiority predictive ability test (CSPA) proposed by Li et al. (2022) to evaluate TMIDAS inflation forecasting methods for the following reasons. Firstly, it is notoriously difficult to forecast inflation and the *unconditional* testing results are not robust in general (e.g. Stock & Watson (2010); Faust & Wright (2013)). Secondly, for empirical forecasters, we are not only interested in evaluating whether one model is better than another *on average*, but also more interested in under what conditioning state this occurs; e.g., in certain conditioning states such as economic expansion or recession. Therefore, as the main contribution in this paper, we are interested to explore whether using a different variable as the conditioning state variable would reveal more exciting information on identifying the model's usefulness, where the *unconditional* test cannot distinguish and fail to capture before. For a given loss function and a conditioning state variable X_t, we consider the following null hypothesis:

$$H_0 : E\left[L_{t+h}\left(y_{t+h}, \hat{f}_{t,j}\right) - L_{t+h}\left(y_{t+h}, \hat{f}_{t,0}\right)|X_t = x\right] \geq 0, 1 \leq j \leq J,$$

where $\hat{f}_{t,j}$ is the forecast estimate of y_{t+h} from model j, and the subscription 0 refers to the benchmark model. Thus, the null hypothesis holds if the benchmark model weakly dominates *all* competing models given our conditioning state X_t. When applying the CSPA test to inflation forecasts, it is initially uncertain which conditioning state variable would provide the most insightful results. This uncertainty arises mainly due to the inherent challenges in inflation forecasting and the numerous macroeconomic variables that could potentially be relevant. Therefore, we examine twelve conditioning state variables that are both important and conceptually distinct. Particularly, we consider the high-frequency index (hfi), the loss differential (ΔL), total inflation (Infl.), Core inflation (Core), inflation differential (Diff), one-month average of the high-frequency variables (PCA, BPCI, TSX, and 3M-yield), and past three-month average of monthly variables (total inflation, core inflation and inflation differentials) at time t.

Our selection of the conditioning state variable is driven by the following considerations. hfi integrates each individual high-frequency indicator over time into a single index, and conditioning on the high-frequency index enables us to incorporate a timely and comprehensive measure of overall economic activity. By conditioning on the loss differential, we can examine whether the future relative performance, assuming two methods perform equally well on average, could have been predicted by current relative performance. Conditioning on the monthly average of high-frequency variables is equivalent to conditioning on various macro-finance indicators. In particular, TSX and BCPI provide insights into the current state of the capital market, while 3M-yield sheds light on the prevailing money market conditions. Considering inflation from the previous month and the average inflation over the past three months, we can use these measures to parsimoniously model how individuals form and anchor their short-term and long-term inflation expectations, assuming no inflation surprises. Test results are presented under column 'CSPA' in Tables 7-11, where each table uses a different high-frequency predictor.

For the one-versus-all CSPA test in the tables, we observe that the test results depend on the forecasting window, the high-frequency predictor, and the conditioning variable used, and the USPA and CSPA results can be different. Overall, all three TMIDAS models can outperform competing models in the majority of cases we considered. In particular, Table 11 shows that the TMIDAS-HFI model with PCA as the high-frequency predictor stands out with superiority performance over other competing models for the one-month-ahead, except for when the past three-month average of inflation differentials is taken as the conditioning state variable. Such superiority property of the TMIDAS-HFI model also holds for various conditioning state variables and other high-frequency predictors.

For the one-versus-one CSPA test, we begin by summarizing the results based on the total number of rejections across *all* conditioning state variables.[9] Our findings indicate that TMIDAS-Core, paired with BCPI as the high-frequency predictor, performs among the best for both one- and three-month-ahead forecasting, with one and three rejections, respectively. Conversely, for the TMIDAS-Diff model, using PCA as the high-frequency predictor yields reasonably good results, with four and ten rejections for one- and three-month-ahead forecasting, respectively. The choice of the high-frequency predictor for the TMIDAS-HFI model depends on the forecasting window. Specifically, using BCPI is favored for one-month-ahead forecasting, resulting in a total rejection of five, while using 3M-yield is preferred for three-month-ahead forecasting, with a total rejection of three. These findings echo those that have been identified in the in-sample results in Section 4.2, in particular for the superior performance of the TMIDAS-HFI model when BCPI and 3M-yield are used for one- and three-month-ahead forecast horizons, respectively.

Next, to explore state-dependent conditional performance, as discussed before, we categorize the twelve conditioning state variables into four groups: the high-frequency-index group, the loss differential group, the macro-finance indicator group and the inflation expectation group. We then summarize the results by counting the number of rejections in each of these groups. For conciseness, we present only the findings that suggest superiority, indicated by zero rejections in each group. First, when evaluating performance conditional on the high-frequency-index value, our results indicate that all three TMIDAS models perform favorably when using PCA as the high-frequency predictor for one-month-ahead forecasting, and 3M-yield as the high-frequency predictor for three-month-ahead forecasting. Secondly, when conditioning on the loss differential, we observe that the TMIDAS-Core model is favored when using PCA and BCPI as the high-frequency predictors for one-month-ahead forecasts, and 3M-yield for three-month-ahead forecasts. The TMIDAS-Diff model and the TMIDAS-HFI model exhibit superiority when utilizing PCA and ER as the high-frequency predictors, respectively, for both one- and three-month-ahead forecasts. Thirdly, when conditioning on various macro-finance indicators, the TMIDAS-Core model stands out as superior for both one- and three-month-ahead forecasts when paired with the predictor PCA and BCPI. Meanwhile, the TMIDAS-Diff model, using PCA and 3M-yield as the predictor, demonstrates the best performance in three-month-ahead forecasting. Moreover, our findings highlight the favorable performance of the TMIDAS-HFI model with ER or TSX for one-month-ahead forecasting, and 3M-yield for three-month-ahead forecasting. Lastly, for inflation forecasting

[9] For comparison purposes, suppose a model is strictly dominated by all other models across all conditioning state variables. In this case, the total rejection number would be $12 \times 9 = 108$.

based on specific inflation expectations, our results indicate that the TMIDAS-Diff model using *PCA* as the high-frequency predictor, exhibits superior performance for one-month-ahead forecasts. In contrast, for three-month-ahead forecasts, both the TMIDAS-Core model and the TMIDAS-HFI model stand out when using 3M-yield among all models.

From the above results, we find that when 3M-yield predictor is used for the three-month-ahead forecasting horizon, at least one of our TMIDAS models has superior performance, regardless of what is selected as the conditioning state variable. This, further to our in-sample results, provide favorable support again to the view in the literature that 3M-yield is a useful predictor and provides strong predictive power for total inflation. In the meantime, for the three out of four groups of conditioning state variables discussed above, our TMIDAS-HFI model consistently outperforms when paired with 3M-yield for three-month-ahead forecasting. For one-month-ahead forecasting, on the other hand, *PCA* remains the winning predictor in all cases whenever our TMIDAS models indicate more competitive performances. This is in line with findings in many studies reporting that more accurate inflation forecasts are obtained by carrying out principal component analysis (see for example, Forni et al. (2003), Eickmeier & Ziegler (2008)).

5 Conclusion

In this paper, we propose threshold mixed data sampling (TMIDAS) models to forecast the annualized monthly total inflation rate in Canada. Our models intend to capture possible non-linearity in inflation forecasting via the threshold effect, in the meantime, to allow higher sampling frequencies for the predictors than the dependent variable. In this paper, we consider different threshold variables including low-frequency core inflation and inflation differentials, and a high-frequency index to split the sample, where the high-frequency index is the weighted average of a high-frequency predictor over time. Both in-sample and out-of-sample performances are evaluated. The in-sample results confirm that our proposed TMIDAS models outperform all considered competing models. Moreover, it achieves the best performance when the high-frequency index is used as the threshold variable. In the out-of-sample forecasting comparison, we conduct USPA tests and CSPA tests with various conditioning state variables as a robustness check. We find no single model has a uniformly best performance over all cases considered in our analysis; however, depending on the choice of the high-frequency predictor, at least one of our TMIDAS models remains to be competitive in most cases compared with other best-performed models in different scenarios.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table 7: **USPA and CSPA Tests with High Frequency Predictor BCPI**

Panel A: One-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	1	1	0	1	4	0	1	3	4	0	0	1	4
	$AR(1)$	0	1	2	0	0	0	0	0	4	0	1	0	0
	Augmented $AR(1)$	0	0	1	0	0	0	0	0	0	0	1	0	0
	ADL	0	0	1	0	0	0	0	0	1	0	1	1	0
	MIDAS	0	0	0	0	0	0	0	0	0	0	0	0	0
	Threshold-$Core$	5	1	2	1	4	0	1	5	3	1	1	4	3
	Threshold-$Diff$	2	2	1	1	0	0	1	0	1	1	0	0	1
	TMIDAS-$Core$	0	0	0	0	0	0	0	1	0	0	0	0	0
	TMIDAS-$Diff$	1	1	1	0	3	1	0	5	1	0	1	0	0
	TMIDAS-HFI	1	1	0	1	1	0	1	0	0	0	0	0	1
One-versus-all	TMIDAS-$Core$		✓				✓							
	TMIDAS-$Diff$	✓	✓				✓		✓					
	TMIDAS-HFI													

Panel B: Three-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	0	0	1	0	0	1	2	1	2	0	0
	$AR(1)$	0	0	1	0	1	0	0	0	2	1	2	1	0
	Augmented $AR(1)$	1	0	1	0	0	1	0	0	0	0	1	0	1
	ADL	0	1	1	0	0	0	1	0	1	1	3	0	0
	MIDAS	1	0	0	0	1	1	0	0	1	0	2	0	3
	Threshold-$Core$	0	0	0	0	0	1	0	0	0	0	0	0	0
	Threshold-$Diff$	0	0	1	3	0	2	3	1	4	0	0	1	2
	TMIDAS-$Core$	1	1	1	0	0	0	0	0	1	0	0	0	0
	TMIDAS-$Diff$	0	0	1	6	0	0	6	0	7	0	3	5	0
	TMIDAS-HFI	6	2	2	7	0	0	7	6	7	9	6	1	2
One-versus-all	TMIDAS-$Core$									✓		✓		
	TMIDAS-$Diff$				✓					✓		✓	✓	
	TMIDAS-HFI	✓								✓	✓	✓		

Notes: This table reports USPA and CSPA test with predictor BCPI. One-versus-one test reports the rejections of the benchmark in each row against each of other nine competing methods. One-versus-all test reports the rejection of the benchmark in each row against *all* other nine competing methods. Symbol ✓ denotes a rejection at the 5% significance level.

Table 8: **USPA and CSPA Tests with High Frequency Predictor TSX**

Panel A: One-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	2	1	0	1	6	0	1	3	0	4	1	0	2
	$AR(1)$	2	5	0	2	1	1	3	1	0	2	2	0	0
	Augmented $AR(1)$	0	0	0	4	0	0	4	0	1	0	0	0	0
	ADL	0	0	0	1	0	0	1	1	0	0	0	0	0
	MIDAS	0	0	0	0	0	0	0	0	0	0	0	0	0
	Threshold-$Core$	4	5	3	5	4	2	4	3	2	3	3	0	2
	Threshold-$Diff$	2	4	4	5	5	5	5	2	3	3	3	3	2
	TMIDAS-$Core$	2	4	2	0	0	0	0	0	0	0	1	0	0
	TMIDAS-$Diff$	2	1	0	4	2	1	4	0	0	0	0	1	4
	TMIDAS-HFI	0	4	1	0	0	0	0	0	0	3	1	0	0
One-versus-all	TMIDAS-$Core$	✓	✓	✓										
	TMIDAS-$Diff$	✓			✓			✓						
	TMIDAS-HFI		✓								✓			

Panel B: Three-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	1	0	0	2	0	0	1	0	1	0	0
	$AR(1)$	0	0	1	0	0	1	0	1	1	0	1	0	1
	Augmented $AR(1)$	0	0	0	0	0	0	0	1	0	0	0	0	0
	ADL	0	1	1	1	0	3	2	2	2	0	0	0	0
	MIDAS	1	0	0	1	0	2	0	1	2	1	1	1	1
	Threshold-$Core$	0	0	0	0	0	0	0	0	0	0	0	0	0
	Threshold-$Diff$	0	1	2	1	0	0	0	1	0	0	1	0	0
	TMIDAS-$Core$	0	0	1	0	0	0	0	0	1	0	0	1	0
	TMIDAS-$Diff$	0	1	2	1	1	1	1	4	1	2	6	0	6
	TMIDAS-HFI	0	1	1	0	5	5	0	0	0	0	0	2	0
One-versus-all	TMIDAS-$Core$		✓							✓		✓	✓	
	TMIDAS-$Diff$		✓						✓	✓	✓	✓	✓	✓
	TMIDAS-HFI		✓				✓			✓		✓	✓	

Notes: This table reports USPA and CSPA test with high frequency predictor TSX. One-versus-one test reports the rejections of the benchmark in each row against each of other nine competing methods. One-versus-all test reports the rejection of the benchmark in each row against *all* other nine competing methods. Symbol ✓ denotes a rejection at the 5% significance level.

Table 9: **USPA and CSPA Tests with High Frequency Predictor ER**

Panel A: One-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	0	0	0	9	0	1	0	3	0	1	0
	$AR(1)$	0	1	0	1	0	0	1	0	0	2	0	0	1
	Augmented $AR(1)$	0	0	0	0	0	0	0	0	0	1	0	0	1
	ADL	0	2	1	1	0	0	0	0	0	1	0	0	1
	MIDAS	0	0	0	0	0	0	0	0	0	1	0	0	1
	Threshold-$Core$	1	1	4	1	0	0	0	0	0	0	6	1	5
	Threshold-$Diff$	4	1	3	4	4	2	4	5	3	7	5	2	3
	TMIDAS-$Core$	3	2	2	0	1	0	0	0	1	3	2	0	4
	TMIDAS-$Diff$	0	1	3	7	0	2	7	0	0	1	0	5	1
	TMIDAS-HFI	0	0	0	0	0	0	0	0	0	1	5	0	0
One-versus-all	TMIDAS-$Core$	✓										✓		
	TMIDAS-$Diff$				✓		✓	✓	✓				✓	✓
	TMIDAS-HFI								✓			✓		

Panel B: Three-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	0	0	0	0	0	0	3	0	0	0	0
	$AR(1)$	0	0	1	1	1	0	1	0	1	1	0	0	0
	Augmented $AR(1)$	3	2	2	0	4	0	1	0	3	1	3	2	1
	ADL	0	0	2	1	1	0	1	0	1	1	0	1	0
	MIDAS	2	1	0	1	3	0	1	1	3	0	1	1	0
	Threshold-$Core$	0	0	0	0	0	0	0	0	0	0	0	5	2
	Threshold-$Diff$	0	0	1	0	2	0	0	0	1	0	0	0	0
	TMIDAS-$Core$	1	1	1	2	3	1	2	0	2	1	1	1	0
	TMIDAS-$Diff$	4	4	1	0	1	4	0	1	4	1	2	1	2
	TMIDAS-HFI	3	1	0	0	1	0	0	0	6	1	0	3	3
One-versus-all	TMIDAS-$Core$				✓				✓				✓	
	TMIDAS-$Diff$	✓									✓	✓		✓
	TMIDAS-HFI			✓				✓					✓	✓

Notes: This table reports USPA and CSPA test with high frequency predictor ER. One-versus-one test reports the rejections of the benchmark in each row against each of other nine competing methods. One-versus-all test reports the rejection of the benchmark in each row against *all* other nine competing methods. Symbol ✓ denotes a rejection at the 5% significance level.

Table 10: **USPA and CSPA Tests with High Frequency Predictor 3M-yield**

Panel A: One-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	1	0	0	5	0	0	0	0	0	1	0
	$AR(1)$	0	0	0	1	0	0	1	0	0	0	1	0	1
	Augmented $AR(1)$	3	5	3	4	0	2	4	0	0	1	1	0	3
	ADL	0	0	0	0	0	0	0	0	0	0	0	0	1
	MIDAS	0	1	0	0	1	0	0	0	2	0	1	0	2
	Threshold-$Core$	1	0	0	0	0	0	0	0	0	1	4	0	1
	Threshold-$Diff$	0	2	1	0	1	1	0	2	2	2	1	1	1
	TMIDAS-$Core$	3	1	1	4	5	4	4	0	7	6	2	3	5
	TMIDAS-$Diff$	0	0	0	1	0	4	0	0	0	1	0	0	0
	TMIDAS-HFI	7	7	3	1	1	6	0	0	2	0	0	0	1
One-versus-all	TMIDAS-$Core$	✓				✓	✓			✓			✓	✓
	TMIDAS-$Diff$						✓							
	TMIDAS-HFI	✓	✓				✓							

Panel B: Three-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	0	0	0	0	0	0	1	0	0	1	0
	$AR(1)$	0	0	1	0	0	0	0	0	2	1	0	1	0
	Augmented $AR(1)$	0	0	0	0	0	0	0	0	0	0	0	0	0
	ADL	1	0	1	1	0	1	1	0	1	2	0	1	0
	MIDAS	0	0	0	0	0	0	0	0	0	0	0	0	0
	Threshold-$Core$	0	0	0	0	0	0	0	0	1	0	0	0	0
	Threshold-$Diff$	0	0	1	0	2	0	0	0	0	2	5	0	2
	TMIDAS-$Core$	0	0	0	6	0	0	6	0	0	0	0	0	0
	TMIDAS-$Diff$	0	0	1	0	0	0	0	5	0	3	5	0	6
	TMIDAS-HFI	0	0	3	0	0	0	0	0	0	0	0	0	0
One-versus-all	TMIDAS-$Core$				✓			✓			✓		✓	✓
	TMIDAS-$Diff$										✓	✓	✓	✓
	TMIDAS-HFI												✓	

Notes: This table reports USPA and CSPA test with high frequency predictor 3M-yield. One-versus-one test reports the rejections of the benchmark in each row against each of other nine competing methods. One-versus-all test reports the rejection of the benchmark in each row against *all* other nine competing methods. Symbol ✓ denotes a rejection at the 5% significance level.

Table 11: **USPA and CSPA Tests with High Frequency Predictor *PCA***

Panel A: One-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	1	0	2	4	1	6	4	2	0	3	0	4	1
	$AR(1)$	2	1	1	4	2	1	4	2	2	3	1	0	3
	Augmented $AR(1)$	0	0	2	0	0	0	0	0	0	0	0	0	0
	ADL	2	1	2	3	2	1	3	2	1	3	1	1	2
	MIDAS	0	0	0	0	0	0	1	0	0	1	0	0	0
	Threshold-$Core$	2	1	2	4	2	2	4	1	1	3	6	2	6
	Threshold-$Diff$	2	2	1	4	4	3	3	4	4	6	4	4	4
	TMIDAS-$Core$	0	0	0	0	0	0	0	0	0	2	0	1	4
	TMIDAS-$Diff$	0	0	0	2	0	0	2	0	0	0	0	0	0
	TMIDAS-HFI	2	0	1	0	0	1	0	0	0	1	0	0	5
One-versus-all	TMIDAS-$Core$										✓		✓	✓
	TMIDAS-$Diff$				✓			✓	✓					
	TMIDAS-HFI													✓

Panel B: Three-month-ahead

Test	Model	USPA	CSPA											
			hfi	ΔL	Mon. Ave.				Infl.	Core	Diff	3M. Ave.		
					PCA	BCPI	TSX	3M-Yield				Infl.	Core	Diff
One-versus-one	HM	0	0	1	0	0	0	0	1	3	0	1	1	0
	$AR(1)$	0	0	2	0	0	0	0	1	2	1	0	1	0
	Augmented $AR(1)$	1	0	2	0	0	0	0	0	2	0	0	0	1
	ADL	0	0	2	0	0	1	0	2	2	0	0	1	1
	MIDAS	0	0	1	0	0	2	0	1	3	1	1	1	1
	Threshold-$Core$	0	0	1	0	0	0	0	0	2	0	0	0	0
	Threshold-$Diff$	0	0	1	0	4	1	0	0	0	3	3	0	1
	TMIDAS-$Core$	0	3	2	0	0	0	0	0	1	2	0	0	2
	TMIDAS-$Diff$	0	0	0	0	0	0	0	1	4	5	0	0	0
	TMIDAS-HFI	0	0	0	2	1	5	2	0	0	2	1	0	0
One-versus-all	TMIDAS-$Core$										✓	✓	✓	✓
	TMIDAS-$Diff$								✓		✓		✓	✓
	TMIDAS-HFI			✓		✓	✓				✓	✓	✓	✓

Notes: This table reports USPA and CSPA test with high frequency predictor PCA. One-versus-one test reports the rejections of the benchmark in each row against each of other nine competing methods. One-versus-all test reports the rejection of the benchmark in each row against *all* other nine competing methods. Symbol ✓ denotes a rejection at the 5% significance level.