Lab 10 Solutions PARTS

March 24, 2017

1 Chapter 11.3.7 *page* 436

Question: Use the total differential to find the MRTS for the production function

$$y = [0.3x_1^{-3} + 0.7x_2^{-3}]^{-\frac{1}{3}} \tag{1}$$

Show that its isoquants are strictly convex to the origin.

Hints:

$$dy = -\frac{1}{3}[0.3x_1^{-3} + 0.7x_2^{-3}]^{-\frac{4}{3}}0.3(-3)x_1^{-4}dx_1 - \frac{1}{3}[0.3x_1^{-3} + 0.7x_2^{-3}]^{-\frac{4}{3}}0.7(-3)x_2^{-4}dx_2 \tag{2}$$

$$dy = [0.3x_1^{-3} + 0.7x_2^{-3}]^{-\frac{4}{3}}0.3x_1^{-4}dx_1 + [0.3x_1^{-3} + 0.7x_2^{-3}]^{-\frac{4}{3}}0.7x_2^{-4}dx_2 \tag{3}$$

Along an isoquant line, dy=0, we have,

$$MRTS = |\frac{dx_2}{dx_1}| = |-\frac{3x_1^{-4}}{7x_2^{-4}}| = |\frac{3x_2^4}{7x_1^4}| \tag{4}$$

Therefore,

$$\frac{d^2x_2}{dx_1^2} = \frac{\partial(-\frac{3x_1^{-4}}{7x_2^{-4}})}{\partial x_1} = -\frac{3x_2^4}{7x_1^5}(-4) = \frac{12x_2^4}{7x_1^5} > 0 \tag{5}$$

$\Rightarrow$ Its isoquants are strictly convex

2 Chapter 11.5.5 *page* 451

Question: Use theorem 11.7 to show that the function

$$y = (x_1 + x_2)^{1/2} \tag{6}$$

defined on $\mathbb{R}^{\nvDash}_{++}$, is a concave function. Show that there are linear segments on the surface of this function and so the function is not strictly concave (see example 11.27).

Theorem 11.7: If the function $y = f(x1, x2)$ defined on $\mathbb{R}^{\nvDash}$ is twice continuously differentiable, then it is concave if and only if

$$d^2y = f_{11}dx_1^2 + 2f_{12}dx_1dx_2 + f_{22}dx_2^2 \leq 0 \tag{7}$$

Hints:

$$f_1 = \frac{1}{2}(x_1 + x_2)^{-\frac{1}{2}} = f_2 \tag{8}$$

$$f_{11} = f_{12} = f_{21} = f_{22} = -\frac{1}{4}(x_1 + x_2)^{-\frac{3}{2}} \tag{9}$$

Therefore,

$$dy = f_{11}dx_1^2 + 2f_{12}dx_1dx_2 + f_{22}dx_2^2 \tag{10}$$

$$= -\frac{1}{4}(x_1 + x_2)^{-\frac{3}{2}}dx_1^2 - \frac{1}{2}(x_1 + x_2)^{-\frac{3}{2}}dx_1dx_2 - \frac{1}{4}(x_1 + x_2)^{-\frac{3}{2}}dx_2^2 \tag{11}$$

$$= -\frac{1}{4}(x_1 + x_2)^{-\frac{3}{2}}(dx_1^2 + 2dx_1dx_2 + dx_2^2) \tag{12}$$

$$= -\frac{1}{4}(x_1 + x_2)^{-\frac{3}{2}}(dx_1 + dx_2)^2 < 0 \tag{13}$$

By theorem 11.7 $\Rightarrow$ it is a concave function.

To show it is not strictly concave, let $x_2 = a - x_1$,

$\Rightarrow y = a^{1/2} \Rightarrow$ there are linear segments on the surface of this function.