Lab 5 Solutions PARTS

February 17, 2017

1 Chapter 7.2

Question: Solve the following pairs of equations by substitution and by elimination.

Hints: We will use substitution method to solve (a), elimination method to solve (b), Gauss Jordan row reduction to solve (c).

(a)

$$y = 24 - x \tag{1}$$
$$2y = 4 + 5x \tag{2}$$

Substitution (1) into (2):

$$2(24 - x) = 4 + 5x \ \Rightarrow x = \frac{44}{7} \ \Rightarrow y = \frac{124}{7} \tag{3}$$

(b)

$$-y = -8x - 4 \tag{4}$$
$$y = 20x + 2 \tag{5}$$

Let (4) + (5), we have:

$$0 = 12x - 2 \ \Rightarrow x = \frac{1}{6} \ \Rightarrow y = \frac{16}{3} \tag{6}$$

(c)

$$0.5y + 2x = 0 \tag{7}$$
$$-y + x = 0 \tag{8}$$

Expressing this in matrix form, we can arrive at the reduced row-echelon form through the following steps:

$$\begin{bmatrix} 0.5 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} \Rightarrow row1 \times 2 \Rightarrow \begin{bmatrix} 1 & 4 & 0 \\ -1 & 1 & 0 \end{bmatrix} \Rightarrow row2 + row1 \Rightarrow \begin{bmatrix} 1 & 4 & 0 \\ 0 & 5 & 0 \end{bmatrix} \Rightarrow$$
$$\Rightarrow row2 \div 2 \Rightarrow \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow row1 - 4 \times row2 \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

From above *row-reduced-echelon* form, we have

$$x = 0\ ,\ \ y = 0 \tag{9}$$

(d)

$$0.5y + 2x = 0 \tag{10}$$

$$-y - 4x = 0 \tag{11}$$

We have **infinite** solution to above 2 by 2 linear equation system. This is because one is linear **dependent** on another. To see this, suppose, we let (10) $\times$ -2, we find the the equation becomes (11).

2 Chapter 7.3

Question: Which of the following systems are linearly dependent and which are inconsistent?

Hints:

(a)

$$2x + y - z = 10 \tag{12}$$

$$4y + 2z = 4 \tag{13}$$

$$x = 0 \tag{14}$$

Substitute (14) into (12):

$$y - z = 10 \;\; \Rightarrow y = 10 + z \tag{15}$$

Now substitute (15) into (13):

$$4(10 + z) + 2z = 4 \;\; \Rightarrow z = -6 \tag{16}$$

Now, we substitute z=-6 back to (13),

$$4y - 12 = 4 \;\; \Rightarrow y = 4 \tag{17}$$

Therefore, we have a unique solution, which can be characterized as following and means that the linear equations are linearly independent with each other and the linear system is consistent.

$$\begin{cases} x = 0 \\ y = 4 \\ z = -6 \end{cases} \tag{18}$$

Consider, instead, if (12) is in form: $2z + y - z = 10$, we may have an inconsistent problem in our linear system.

(b)

$$-y + z = 0 \tag{19}$$

$$4x + 2y - z/3 = 0 \tag{20}$$

$$x + z = 0 \tag{21}$$

Now, subtract (21) from (19),

$$x + y = 0 \tag{22}$$

Let (21) be divided by 3 both sides and plus (20),

$$\frac{x}{3} + \frac{z}{3} + 4x + 2y - \frac{z}{3} = 0 \ \Rightarrow \frac{13}{3}x + 2y = 0 \ \Rightarrow \frac{13}{6}x + y = 0 \tag{23}$$

Now, subtract (23) from (22),

$$\frac{7}{6}x = 0 \ \Rightarrow x = 0 \tag{24}$$

Now substitute x=0 back to (22) & (21)

$$y = 0 \ , \ z = 0 \tag{25}$$

Therefore, we have a unique solution of this linear system:

$$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \tag{26}$$

(c)

$$-3x + 2y - z = 14 \tag{27}$$

$$-x - y - z = 0 \tag{28}$$

$$x + 10y - 3z = 2 \tag{29}$$

Let (28) plus (29):

$$-x - y - z + x + 10y - 3z = 2 \ \Rightarrow 9y - 4z = 2 \ \Rightarrow z = \frac{9y - 2}{4} \tag{30}$$

Now, let (28) $\times$ 3 and plus (27):

$$3x + 30y - 9z - 3x + 2y - z = 6 + 14 = 20 \ \Rightarrow 32y - 10z = 20 \tag{31}$$

Now, substitute (30) into (31),

$$32y - 10 \times \frac{9y - 2}{4} = 20 \ \Rightarrow y = \frac{30}{19} \tag{32}$$

Substitute (32) back to (30),

$$z = \frac{58}{19} \tag{33}$$

Substitute (33) & (32) into (28),

$$x = -\frac{88}{19} \tag{34}$$

Therefore, we have a unique solution of this linear system:

$$\begin{cases} x = -\frac{88}{19} \\ y = \frac{30}{19} \\ z = \frac{58}{19} \end{cases} \tag{35}$$

3 Chapter 7.4

Question: An economy has three markets with supply and demand functions for the three goods given by:

$$q_1^s = -20 + p_1 - 0.5p_2 \tag{36}$$
$$q_2^s = -100 + 2p_2 \tag{37}$$
$$q_3^s = p_3 \tag{38}$$

$$q_1^d = 80 - 2p^1 - p^3 \tag{39}$$
$$q_2^d = 200 - p_2 \tag{40}$$
$$q_3^d = 100 - 2p_3 - p_1 \tag{41}$$

3.1

Comment on the relationship between the three goods on the demand side.

Hints: From (39), we find that with good 3 price increases, good 1 demand decreases. Similarly, from (41), we find that with good 1 price increases, good 3 demand decreases. We can determine that good 1 and good 3 are **complementary goods**. (40) shows that demand on good 2 is only determined ny good 2 price, which implies that good 2 is neither substitute nor complement of good 1 and good 3.

3.2

Solve for the equilibrium prices and quantities of the three goods.

At market equilibrium, we must have,

$$q_1^s = q_1^d \Rightarrow -20 + p_1 - 0.5p_2 = 80 - 2p^1 - p^3 \Rightarrow 3p_1 - 0.5p_2 - p^3 = 100 \tag{42}$$

$$q_2^s = q_2^d \Rightarrow -100 + 2p_2 = 200 - p_2 \Rightarrow 3p2 = 300 \tag{43}$$

$$q_3^s = q_3^d \Rightarrow p_3 = 100 - 2p_3 - p_1 \Rightarrow p_1 + 3p_3 = 100 \tag{44}$$

Expressing this in matrix form, we can arrive at the reduced row-echelon form through the following steps:

$$\begin{bmatrix} 3 & -1/2 & 1 & 100 \\ 0 & 3 & 0 & 300 \\ 1 & 0 & 3 & 100 \end{bmatrix} Row1/3 \ \& \ Row2/3 \Rightarrow \begin{bmatrix} 1 & -1/6 & 1/3 & 100/3 \\ 0 & 1 & 0 & 100 \\ 1 & 0 & 3 & 100 \end{bmatrix} Row1 \ + \ Row2/6 \ \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 1/3 & 50 \\ 0 & 1 & 0 & 100 \\ 1 & 0 & 3 & 100 \end{bmatrix} Row3 \ - \ Row1 \ \Rightarrow \begin{bmatrix} 1 & 0 & 1/3 & 50 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 10/3 & 50 \end{bmatrix} Row3 \ \times \ 3/10 \ \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 1/3 & 50 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 75/4 \end{bmatrix} Row1 \ + \ Row3/3 \ \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 175/4 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 75/4 \end{bmatrix}$$

Therefore, the equilibrium price for good 1 is 175/4, the equilibrium price for good 2 is 100 & the equilibrium price for good 3 is 75/4.

Now, substitute three equilibrium prices back to (36), (37) & (38), we can calculate the equilibrium quantities.