Lab 6 Solutions PARTS

February 17, 2017

1 Chapter 8.5

Question: Let

$$A = \begin{bmatrix} 3 & -4 \\ -5 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 7 & 4 \\ 5 & k \end{bmatrix}$$

What values of k, if any, will make $AB = BA$?

Hints:

$$AB = \begin{bmatrix} 3 & -4 \\ -5 & 1 \end{bmatrix} * \begin{bmatrix} 7 & 4 \\ 5 & k \end{bmatrix} = \begin{bmatrix} 21-20 & 12-4k \\ -35+5 & -20+k \end{bmatrix} = \begin{bmatrix} 1 & 12-4k \\ -30 & -20+k \end{bmatrix}$$

$$BA = \begin{bmatrix} 7 & 4 \\ 5 & k \end{bmatrix} * \begin{bmatrix} 3 & -4 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 21-20 & -28+4 \\ 15-5k & -20+k \end{bmatrix} = \begin{bmatrix} 1 & -24 \\ 15-5k & -20+k \end{bmatrix}$$

If $AB = BA$, we must have:

$$\begin{cases} 12-4k = -24 \\ 15-5k = -30 \\ -20+k = -20+k \end{cases} \Rightarrow k = 9 \tag{1}$$

Thus, $k = 9$ gives us $AB = BA$.

2 Chapter 8.6

Question: Compute the quantities below using

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}, \ B = \begin{bmatrix} -8 & 4 \\ -7 & 5 \end{bmatrix}$$

(a) A^T, B^T, $A^T + B^T$, $(A+B)^T$

(b) AB, $(AB)^T$, $A^T B^T$, $B^T A^T$

Hints:

(a)

$$A^T = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -8 & -7 \\ 4 & 5 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 1-8 & -3-7 \\ 2+4 & 4+5 \end{bmatrix} = \begin{bmatrix} -7 & -10 \\ 6 & 9 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 1-8 & 2+4 \\ -3-7 & 4+5 \end{bmatrix}^T = \begin{bmatrix} -7 & 6 \\ -10 & 9 \end{bmatrix} = \begin{bmatrix} -7 & -10 \\ 6 & 9 \end{bmatrix}$$

(b)

$$AB = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} * \begin{bmatrix} -8 & 4 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} -8-14 & 4+10 \\ 24-28 & -12+20 \end{bmatrix} = \begin{bmatrix} -22 & 14 \\ -4 & 8 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} -22 & -4 \\ 14 & 8 \end{bmatrix}$$

$$A^T B^T = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} * \begin{bmatrix} -8 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -8-12 & -7-15 \\ -16+16 & -14+20 \end{bmatrix} = \begin{bmatrix} -20 & -22 \\ 0 & 6 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} -8 & -7 \\ 4 & 5 \end{bmatrix} * \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -8-14 & 24-28 \\ 4+10 & -12+20 \end{bmatrix} = \begin{bmatrix} -22 & -4 \\ 14 & 8 \end{bmatrix}$$

3 Chapter 8.8

Question: Use the data in example 8.11 to find the unemployment rate after two periods. [Hint: The unemployment rate is the number of people unemployed as a proportion of all labor market participants.] Can the situation described by these transition probabilities evolve in the same way indefinitely?

Example 8.11: Labor Market Conditions after One Period

Suppose the labor market transition probability matrix is:

$$P = \begin{bmatrix} 0.8 & 0.1 & 0.01 \\ 0.15 & 0.6 & 0.49 \\ 0.05 & 0.3 & 0.5 \end{bmatrix}$$

and the initial distribution of individuals (in millions) is

$$X^0 = \begin{bmatrix} 10 \\ 1 \\ 5 \end{bmatrix}$$

Hints: the labor market status vector after one period is:

$$X^1 = PX^0 = \begin{bmatrix} 0.8 & 0.1 & 0.01 \\ 0.15 & 0.6 & 0.49 \\ 0.05 & 0.3 & 0.5 \end{bmatrix} * \begin{bmatrix} 10 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8.15 \\ 4.55 \\ 3.3 \end{bmatrix}$$

From textbook (3^{rd} edition) page 286:

*"Note that the increase in unemployment comes both from a net reduction in the number of nonparticipants and a reduction in employment. A situation in which an increase in unemployment is accompanied by an increase in participation is referred to as the **added-worker effect**."*

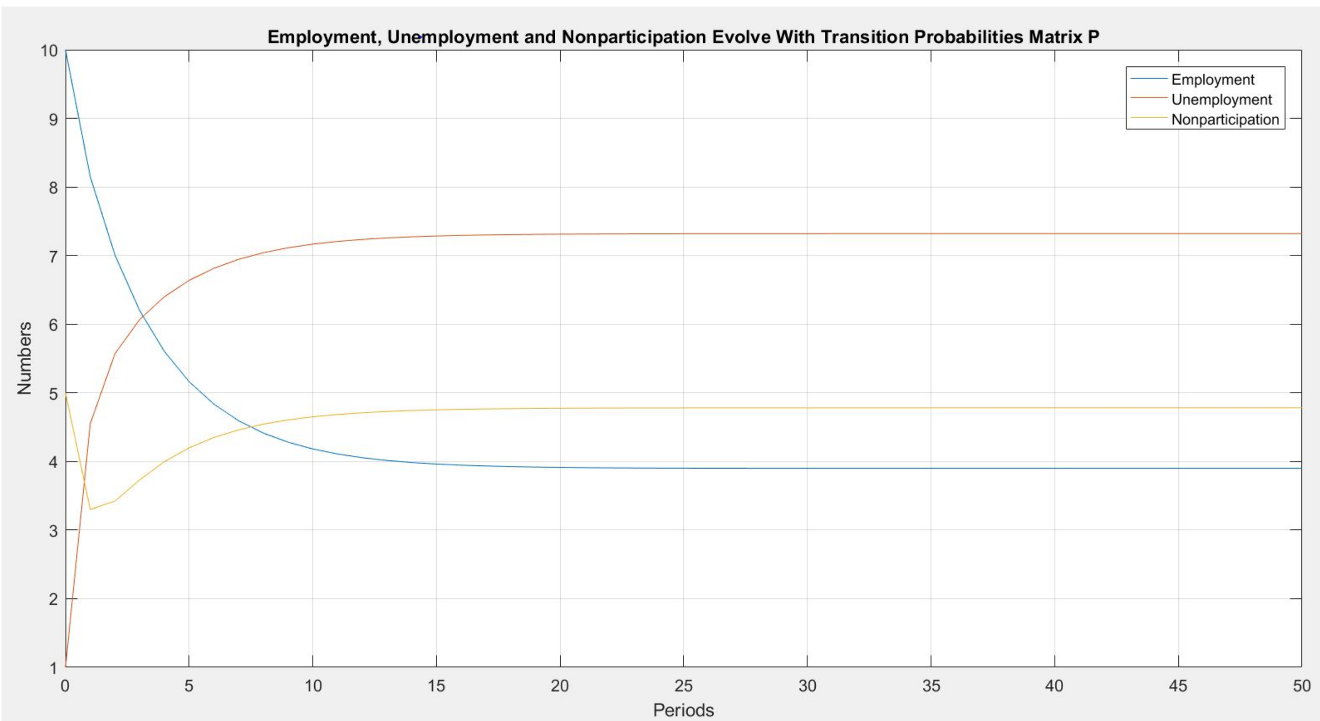
the labor market status vector after two periods is:

$$X^2 = PX^1 = P^2X^0 = \begin{bmatrix} 0.8 & 0.1 & 0.01 \\ 0.15 & 0.6 & 0.49 \\ 0.05 & 0.3 & 0.5 \end{bmatrix} * \begin{bmatrix} 8.15 \\ 4.55 \\ 3.3 \end{bmatrix} = \begin{bmatrix} 7.008 \\ 5.5695 \\ 3.4225 \end{bmatrix}$$

From X^2, we find that unemployment still increases and employment still decreases comparing with X^1 . However, instead of reduction, there is a net increment in the number of nonparticipants.

For more periods evolve situation, Figure 1 shows the detail:

Figure 1


Employment, Unemployment and Nonparticipation Evolve With Transition Probabilities Matrix P
Employment
Unemployment
Nonparticipation
Numbers
Periods
10
9
8
7
6
5
4
3
2
1
0
5
10
15
20
25
30
35
40
45
50