Lab 7 Solutions PARTS

March 3, 2017

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Question: Find the determinant of B:

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{bmatrix}$$

if

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

Hints:
By *Laplace* expansion, we have:

$$\begin{aligned} |A| &= (-1)^{3+1} g \begin{vmatrix} b & c \\ e & f \end{vmatrix} + (-1)^{3+2} h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + (-1)^{3+3} i \begin{vmatrix} a & b \\ d & e \end{vmatrix} \\ &= g \begin{vmatrix} b & c \\ e & f \end{vmatrix} - h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + i \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{aligned}$$

$$\begin{aligned} |B| &= (-1)^{3+1} 3g \begin{vmatrix} b & c \\ e & f \end{vmatrix} + (-1)^{3+2} 3h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + (-1)^{3+3} 3i \begin{vmatrix} a & b \\ d & e \end{vmatrix} \\ &= 3(g \begin{vmatrix} b & c \\ e & f \end{vmatrix} - h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + i \begin{vmatrix} a & b \\ d & e \end{vmatrix}) \\ &= 3\,|A| = 21 \end{aligned}$$

Property 1: If we multiply a scalar to a matrix A, then the value of the determinant will change by a factor.

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Question: Find the determinant of C:

$$C = \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$$

if

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$$

Hints: By *Laplace* expansion, we have:

$$\begin{aligned} |A| &= (-1)^{1+1} a \begin{vmatrix} e & f \\ h & i \end{vmatrix} + (-1)^{1+2} b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + (-1)^{1+3} c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ &= a(ei - fh) - b(di - gf) + c(dh - eg) \\ &= aei - afh - bdi + bgf + cdh - ceg \end{aligned}$$

$$\begin{aligned} |C| &= (-1)^{1+1} a \begin{vmatrix} h & i \\ e & f \end{vmatrix} + (-1)^{1+2} b \begin{vmatrix} g & i \\ d & f \end{vmatrix} + (-1)^{1+3} c \begin{vmatrix} g & h \\ d & e \end{vmatrix} \\ &= a(hf - ei) - b(gf - di) + c(ge - dh) \\ &= ahf - aei - bgf + bdi + cge - cdh \\ &= -|A| \\ &= -3 \end{aligned}$$

Property 2: the sign of the determinant will change if you interchange two rows.

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Question: Find the determinant of D:

$$D = \begin{bmatrix} a & b & c \\ 3d+a & 3e+b & 3f+c \\ g & h & i \end{bmatrix}$$

if

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$$

Hints: By *Property 1*, we must have:

$$\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} = 3\,|A| = 9$$

Then, by *Laplace* expansion:

$$|D| = \begin{vmatrix} a & b & c \\ 3d+a & 3e+b & 3f+c \\ g & h & i \end{vmatrix} = (-1)^{2+1}(3d+a)\begin{vmatrix} b & c \\ h & i \end{vmatrix} + (-1)^{2+2}(3e+b)\begin{vmatrix} a & c \\ g & i \end{vmatrix} + (-1)^{2+3}(3f+c)\begin{vmatrix} a & b \\ g & h \end{vmatrix}$$

$$= (-1)^{2+1}3d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + (-1)^{2+2}3e\begin{vmatrix} a & c \\ g & i \end{vmatrix} + (-1)^{2+3}3f\begin{vmatrix} a & b \\ g & h \end{vmatrix} + (-1)^{2+1}a\begin{vmatrix} b & c \\ h & i \end{vmatrix} + (-1)^{2+2}b\begin{vmatrix} a & c \\ g & i \end{vmatrix} + (-1)^{2+3}c\begin{vmatrix} a & b \\ g & h \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} + (-1)^{2+1}a\begin{vmatrix} b & c \\ h & i \end{vmatrix} + (-1)^{2+2}b\begin{vmatrix} a & c \\ g & i \end{vmatrix} + (-1)^{2+3}c\begin{vmatrix} a & b \\ g & h \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} = 9$$

Where, the last three terms **vanished** since the they are from multiplying the elements of the **first** row by the cofactor of the **second** row.

Therefore, we find:

Property 3: Adding a row/column to another or other elementary row operations will not affect the value of the determinant.