Lab 9 Solutions PARTS

March 17, 2017

1 Chapter 11.1.9 *page* 407

Question: Find the marginal-product functions for the CES (constant elasticity of substitution) production function:

$$y = 12[0.4x_1^{-\frac{1}{2}} + 0.6x_2^{-\frac{1}{2}}]^2 \tag{1}$$

Hints:

$$MP_1 = \frac{\partial y}{\partial x_1} = -2 \times 12 \times [0.4x_1^{-\frac{1}{2}} + 0.6x_2^{-\frac{1}{2}}]^{-3} \times (-\frac{1}{2}) \times 0.4 \times x_1^{-\frac{3}{2}} \tag{2}$$

$$= 4.8 \times [0.4x_1^{-\frac{1}{2}} + 0.6x_2^{-\frac{1}{2}}]^{-3} x_1^{-\frac{3}{2}} \tag{3}$$

$$MP_2 = \frac{\partial y}{\partial x_1} = -2 \times 12 \times [0.4x_1^{-\frac{1}{2}} + 0.6x_2^{-\frac{1}{2}}]^{-3} \times (-\frac{1}{2}) \times 0.6 \times x_2^{-\frac{3}{2}} \tag{4}$$

$$= 7.2 \times [0.4x_1^{-\frac{1}{2}} + 0.6x_2^{-\frac{1}{2}}]^{-3} x_2^{-\frac{3}{2}} \tag{5}$$

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Question: For the function $f(x_1, x_2) = x_1^3 x_2^4$, determine the first- and second-order partial derivatives, and arrange in vector/matrix notation.

Hints:

$$\bigtriangledown f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 3x_1^2 x_2^4 \\ \\ 4x_1^3 x_2^2 \end{bmatrix}$$

$$\bigtriangledown_2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 6x_1 x_2^4 & 12x_1^2 x_2^3 \\ \\ 12x_1^2 x_2^3 & 12x_1^3 x_2^2 \end{bmatrix}$$

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Question: Complete the exercise begun in example 11.11. That is, for the function:

$$f(x_1, x_2, x_3) = x_1^2 e^{3x_2 + x_1 x_3} + 2x_2^3/x_1 \tag{6}$$

show that $f_{12} = f_{21}$, and $f_{13} = f_{31}$

Hints:

$$f_1 = 2x_1 e^{3x_2 + x_1 x_3} + x_1 e^{3x_2 + x_1 x_3} x_3 - 2x_2^3/x_2^2 \tag{7}$$

$$f_2 = (x_1^2) 3e^{3x_2 + x_1 x_3} + 6x_2^2/x_1 \tag{8}$$

$$f_3 = (x_1^3) e^{3x_2 + x_1 x_3} \tag{9}$$

$$f_{12} = 2x_1 3e^{3x_2 + x_1 x_3} + 3x_1 x_3 e^{3x_2 + x_1 x_3} - 6x_2^2/x_1^2 = 6x_1 e^{3x_2 + x_1 x_3} + 3x_1 x_3 e^{3x_2 + x_1 x_3} - 6x_2^2/x_1^2 \tag{10}$$

$$f_{21} = 2x_1 3e^{3x_2 + x_1 x_3} + (x_1^2) 3e^{3x_2 + x_1 x_3} - 6x_2^2/x_1^2 = 6x_1 e^{3x_2 + x_1 x_3} + 3x_1 x_3 e^{3x_2 + x_1 x_3} - 6x_2^2/x_1^2 = f_{12} \tag{11}$$

$$f_{13} = 2x_1 e^{3x_2 + x_1 x_3} x_1 + x_1^2 e^{3x_2 + x_1 x_3} + x_1^2 e^{3x_2 + x_1 x_3} x_1 x_3 \tag{12}$$

$$= 2x_1^2 e^{3x_2 + x_1 x_3} + x_1^2 e^{3x_2 + x_1 x_3} + x_3 x_3 e^{3x_2 + x_1 x_3} \tag{13}$$

$$= 3x_1^2 e^{3x_2 + x_1 x_3} + x_1^3 x_3 e^{3x_2 + x_1 x_3} \tag{14}$$

$$f_{31} = 3x_1^2 e^{3x_2 + x_1 x_3} + x_1^3 x_3 e^{3x_2 + x_1 x_3} = f_{13} \tag{15}$$